

# Stochastic Processes I

Prerequisites for the lectures are a course in basic probability theory and basic knowledge in measure theory. The lectures offer an introduction into the theory of stochastic processes with indepth study of some fundamental classes of stochastic processes and their applications.

## Contents

1. Definition of stochastic processes, construction and existence, classification.  
Kolmogorov and Ionescu-Tulcea theorems.
2. Markov chains with discrete time and discrete state space.  
Classification, stopping times and strong Markov property, asymptotic behaviour and limiting distributions, equilibrium behaviour and stationary distributions.
3. Markov processes with continuous time and discrete state space.  
Classification, analytical properties of the transition semigroup, infinitesimal generator, backward and forward differential equation, explosion in finite time, stopping times and strong Markov property, asymptotic behaviour and limiting distributions, equilibrium behaviour and stationary distributions. Birth-death processes.
4. Martingales with discrete time.  
Definition of martingales, sub- and supermartingales, stopping times, optimal sampling and optional stopping, limit theorems, representation of uniformly integrable martingales.
5. Stochastic processes with stationary and independent increments (Levy processes).  
General properties, infinitely divisible distributions, Poisson process, Brownian motion.
6. General Markov processes and Markovian semigroups.  
Semigroups associated with a Markov process. Infinitesimal generator and differential equations, convolution semigroups, the problem of continuity and differentiability of paths.
7. Renewal theory and renewal processes.  
Counting processes, renewal processes, renewal function, renewal measure, renewal equation, residual lifetime process, renewal paradoxon, general regenerative processes, limit theorems, limiting distributions for age processes.

## References

- [Asm87] S. Asmussen. *Applied Probability and Queues*. John Wiley and Sons, Inc., Chichester – New York – Brisbane – Toronto – Singapore, 1987.
- [Bre68] L. Breiman. *Probability*. Addison–Wesley Publishing Company, Reading, Massachusetts – Menlo Park, California – London – Don Mills, Ontario, 1968.
- [Chu67] K. L. Chung. *Markov Chains with Stationary Transition Probabilities*. Springer–Verlag, Berlin – Heidelberg – New York, 1967.
- [KT75] S. Karlin and H. M. Taylor. *A First Course in Stochastic Processes*. Academic Press, New York – San Francisco – London, second edition, 1975.
- [KT81] S. Karlin and H. M. Taylor. *A Second Course in Stochastic Processes*. Academic Press, New York – San Francisco – London, 1981.

- [Res92] S. Resnick. *Adventures in Stochastic Processes*. Birkhäuser, Boston, 1992.
- [RSST99] T. Rolski, H. Schmidli, V. Schmidt, and J. Teugels. *Stochastic Processes for Insurance and Finance*. Wiley Series in Probability and Statistics. Wiley, Chichester, 1999.