Infinite matroid theory exercise sheet 10

1. Is there a finite field k such that every finite uniform matroid is representable over k?

2. Is there a finite field k such that every frame matroid is thin sums representable over k? The frame matroids are defined in exercise 4 at sheet 5.

3.* Find infinitely many matroids such that none of them is a minor of another. You can find them such that they are all finite, have rank 3 and are representable over $\mathbb{R}$.

4.* Let M be a connected infinite matroid such that all its cocircuits are finite. Let e be an element of the ground set E of M. Using the compactness lemma, prove that there is a subset C of E containing e and such that C does not include any finite circuit of M or meet any corcircuit of M just once. Deduce that M has an infinite circuit. Show that any matroid all of whose circuits and cocircuits are finite must be a direct sum of finite matroids.

5.** Is there a non-tame binary matroid?

Hints

Do exercise 4. Concerning exercise 2: Try to apply exercise 1.