Infinite matroid theory exercise sheet 4

1. Show that if $\mathcal{C}$ satisfies infinite circuit elimination $(C3)_\infty$, then so does $\mathcal{C}^*$.

2. Let X be some uncountable set, and let $y \notin X$. Let $\mathcal{I}$ consist of those subsets of $X + y$ that either are countable or do not contain y. Show that $\mathcal{I}$ satisfies (I1)-(I3) but there is no scrawl system whose set of independent sets is $\mathcal{I}$.

3. Let G be a graph. Let $\mathcal{C}$ be the collection of edge sets of thetas, handcuffs, degenerate handcuffs, double rays and sperms in G, see Figure 1 and Figure 2. For every tree T, let ∂T consist of those edges not in T that have at least one endvertex in T. Let $\mathcal{D}$ consist of all sets ∂T for (not necessary spanning) rayless trees T in G. Show that the pair $(\mathcal{C}, \mathcal{D})$ satisfies (01) and (02). Deduce that $\mathcal{C}$ satisfies$(C3)_\infty$.

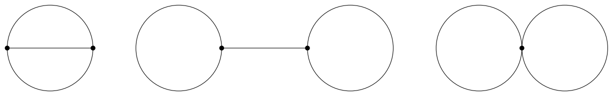

Figure 1: A *theta* is a subdivision of the graph on the left. A *handcuff* is a subdivision of the graph in the middle. A *degenerate handcuff* is a subdivision of the graph on the right.

Figure 2: A *double ray* is the graph on the left. A *sperm* is a subdivision of a the graph on the right.

4.* Let G be a graph that has no subdivision of the Bean graph. Show that $|C \cap D|$ is finite for every $C \in \mathcal{C}_{AC}(G)$ and every $D \in \mathcal{D}_{AC}(G)$ such that at least one rayless side of D is connected.

5.* Let G be a bipartite graph where all vertices of the left bipartition-class have finite degree. Show that there is a scrawl system whose ground set is the set of vertices on the left, and whose independent sets are precisely the sets of vertices on the left that can be matched into the right hand side.

Hints

Concerning question 4:

Remember the proof that $C \in \mathcal{C}_{AC}(G)$ and $D \in \mathcal{D}_{AC}(G)$ satisfy (02).

Concerning question 5:

A compactness argument might be needed.