Infinite matroid theory exercise sheet 5

1. Let M be a matroid. Let $\mathcal{C}$ be the set of finite circuits of M. Show that $\mathcal{C}$ is the set of circuits of some matroid. This matroid is called the *finitarisation* M^{fin} of M.

2. Let M be a matroid. Is it true that if some base of M is a base of M^{fin}, then $M = M^{\text{fin}}$?

3. Characterise for which graphs G, the algebraic cycle matroid $M_A(G)$ is cofinitary.

4. Let G be a graph. Let $\mathcal{C}$ be the collection of edge sets of thetas, handcuffs, degenerate handcuffs, double rays and sperms in G, see Figure 1 and Figure 2. Prove that $\mathcal{C}$ is the set of circuits of some matroid (You may use exercise 3 from sheet 4).

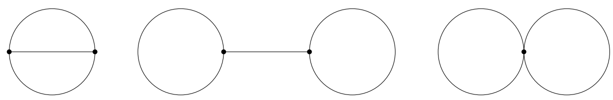

Figure 1: A *theta* is a subdivision of the graph on the left. A *handcuff* is a subdivision of the graph in the middle. A *degenerate handcuff* is a subdivision of the graph on the right.

Figure 2: A *double ray* is the graph on the left. A *sperm* is a subdivision of a the graph on the right.

5. Let M be a finitary matroid, and N be a finite minor of M. Prove that there is a finite set C and some set D such that $N = M/C \backslash D$.

6.** Let M_f and M_c be a finitary matroid and a cofinitary matroid, respectively. Assume that $M_c^{\text{fin}} = M_f$, and that $(M_f^*)^{\text{fin}} = M_c^*$. Let M be a matroid such that $M^{\text{fin}} = M_f$, and that $(M^*)^{\text{fin}} = M_c^*$.

   Let F be some finite matroid. Is it true that if M_f and M_c do not have N as a minor, then M does not have N as a minor? Already for $F = U_{2,4}$ or $F = M(K_4)$ we do not know the answer.

Hints

Concerning question 4:

Suppose that $\mathcal{C}$ is the set of circuits of some matroid M. What do the minors of M look like?