

Infinite matroid theory exercise sheet 7

1. Find a graph-theoretic proof that any 2-connected finitely separable graph is countable.
2. Prove that a multigraph G is 3-connected if and only if it is simple and the finite cycle matroid $M_{FC}(G)$ is 3-connected.
3. Find nontrivial sufficient conditions on G for the topological cycle matroid $M_{TC}(G)$ to be 3-connected.
4. Let M and N be two matroids such that $M^{\text{fin}} = N$ and $(N^*)^{\text{fin}} = M^*$. Show that each k -separation of M is a k -separation of N , and vice versa.
- 5*. For any matroid M , let $\mathcal{F}(M)$ be the smallest set containing M that is closed under duality and finitarisation. Find a matroid M such that $|\mathcal{F}(M)|$ is maximal.
- 6*. Let M be a connected matroid such that every circuit of M and every cocircuit of M is countable. Prove that M is countable.

Hints

Concerning exercise 6: It might be helpful to think about fundamental circuits and cocircuits.