$M \rightsquigarrow M[G]$

$M \models ZFC \rightsquigarrow M[G] \models ZFC$

M can "talk" about truth in $M[G]$ via the forcing relation $\Vdash^*$

Equiv: V = universe. $\Vdash^*$ forcing relation

$\mathbb{P}$ ..... $p \in \mathbb{P}$.

$p \Vdash^* \varphi \Rightarrow$ "$V[G] \models \varphi$"

$V \rightsquigarrow V[G]$

$M \rightsquigarrow M[G]$

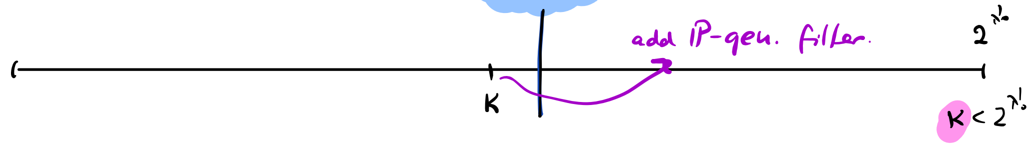
- One forcing extension: $M \to M[G]$
  $G \cap D \neq \emptyset \quad \forall$ dense $D \in M$.

- MA_κ: $\forall \mathbb{P}$ ccc ... $\mathcal{D} = \{D_\alpha : \alpha < \kappa\}$ $\exists G$ s.t. $G \cap D_\alpha \neq \emptyset$ $\forall \alpha < \kappa$.

- Iterate "all possible" ccc forcing notions:

$N = M[G_\kappa]$

$M \subseteq M[G_0] \subseteq M[G_0][G_1] \subseteq \ldots M[G_\alpha]$ $M \subseteq [\ldots G_\alpha \ldots]$ ......



N :: a lot of forcing has been done.

$\mathbb{P}$, $\{D_\beta : \beta < \kappa\}$ $\mathbb{P}$-dense sets.

.... (arg.) .... $\Longrightarrow$ $\mathbb{P}$, $\{D_\beta : \beta < \kappa\} \subseteq M[G_\alpha]$

Some $\gamma > \alpha$, I will add a $M[G_\alpha]$-generic G_γ

which $G_\gamma \cap D_\beta$ $(\beta < \kappa)$. $\Longrightarrow$ MA_κ