Die Komplexen Zahlen

$$x^2 + 1 = 0$$

$$i^2 + 1 = 0 \quad , i^2 = -1 \quad , i = \sqrt{-1}$$

$$\underline{z = x + iy} \quad , \quad x, y \in \mathbb{R}$$

$$\operatorname{Re} z = x \qquad \operatorname{Im} z = y$$

$$z = (x, y) \quad \underline{\in \mathbb{R}^2}$$

$$\underline{(x_1, y_1)} + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$(x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$$

$$(0, 0) \qquad (-x_1, -y_1)$$

$$(x_1, y_1) + (-x_1, -y_1) = (0, 0)$$

$$-1 = i^2 = \sqrt{-1}\cdot\sqrt{-1} = \sqrt{(-1)^2} = 1$$

$$(x_1, y_1)\cdot(x_2, y_2) = (x_1 x_2 - y_1 y_2,\ x_1 y_2 + x_2 y_1)$$

$$(1, 0)$$

$$(x_1, y_1)\cdot(1, 0) = (x_1, y_1)$$

$$z^{-1} = \left(\frac{x}{x^2+y^2},\ \frac{-y}{x^2+y^2}\right)$$

$$z\cdot z^{-1} = (1, 0)$$

$$(x_1, 0) + (x_2, 0) = (x_1 + x_2, 0)$$

$$(x_1, 0) \cdot (x_2, 0) = (x_1 x_2, 0)$$

$$\mathbb{R} \to \mathbb{C}, \; x \mapsto (x, 0)$$

$$\mathbb{C} \setminus \{0\} =: \mathbb{C}^\times$$

$(1,0)$ $\qquad z = x + iy$

$$(0,1)^2 = (-1,0)$$

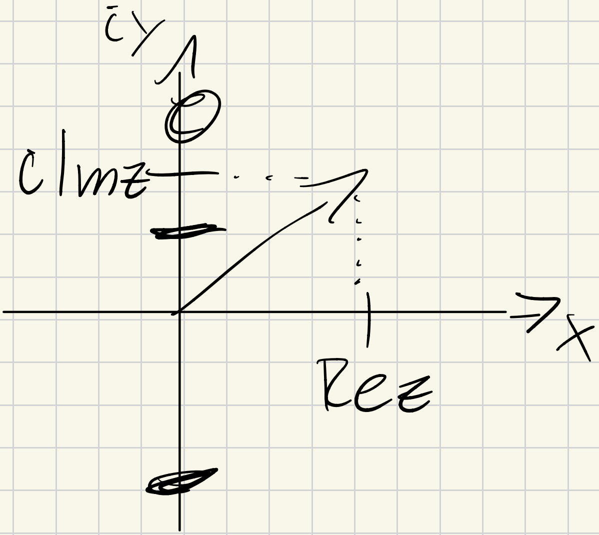
Basis: $\{(1,0),(0,1)\}$

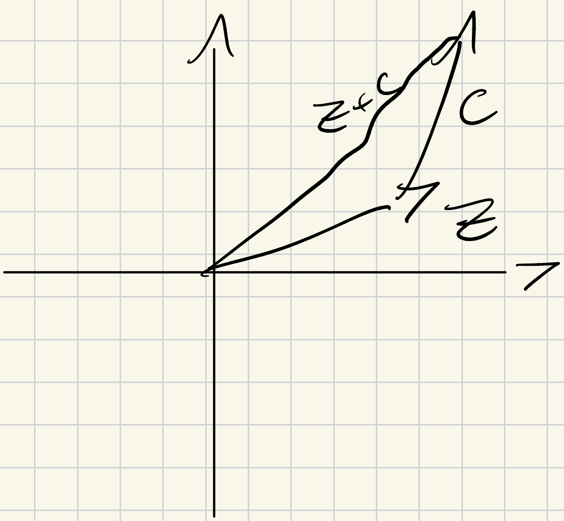
$(x_1,y_1)\cdot(x_2,y_2)$

$[x_1\cdot(1,0)+y_1(0,1)][x_2(1,0)+y_2(0,1)]$

$x_1x_2(1,0)+x_1\cdot y_2(0,1)+y_1x_2(0,1)+y_1y_2(0,1$

$(x_1x_2-y_1y_2)(1,0)+(x_1y_2+y_1x_2)(0,1)$


iy
x
i Im z
Re z


z+c
c
z

„>0" des positiv seins

$z>0 \vee z=0 \vee -z>0$

$z>0,\ c>0 \Rightarrow \underline{z+c>0},\ z\cdot c>0$

$i \neq 0 \qquad i>0 \qquad i^2>0$

$\qquad\qquad -i>0 \qquad (-i)^2 = i^2 > 0$

$\underbrace{1+i^2}_{=0} > 0 \qquad$ ↯ $\qquad i^2=-1$

Darstellung als 2x2 Matrix

$c = a + ib$, $z = x + iy$

$T_c : \mathbb{C} \to \mathbb{C}$, $z \mapsto cz = \underbrace{ax - by} + i(\underbrace{ay + bx})$

$$T_c\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax - by \\ ay + bx \end{pmatrix} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$F : \mathbb{C} \to \mathrm{Mat}(2, \mathbb{R})$

$$c = a + ib \mapsto \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

$F(c \cdot z) = F(c) \cdot F(z)$

$F(c + z) = F(c) + F(z)$

$$\left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \;\middle|\; a, b \in \mathbb{R} \right\}$$

$$\mathbb{C} \to \mathbb{C}, \quad z \mapsto \bar{z}$$

$$z = x + iy, \quad \bar{z} := x - iy$$

$$F(c) = \begin{pmatrix} \underline{a} & -b \\ b & \underline{a} \end{pmatrix} \quad F(\bar{c}) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

$$\overline{cz} = \bar{c}\,\bar{z}$$

$$\forall a \in \mathbb{R}: \ \bar{a} = a$$

$w = u + iv, \quad z = x + iy \qquad a \in \mathbb{R}$

$\langle w, z \rangle := \operatorname{Re}(w\bar{z}) = ux + vy$

$\langle w + w', z \rangle = \langle w, z \rangle + \langle w', z \rangle$

$\langle aw, z \rangle = a \langle w, z \rangle$

$\langle w, z \rangle = \langle z, w \rangle$

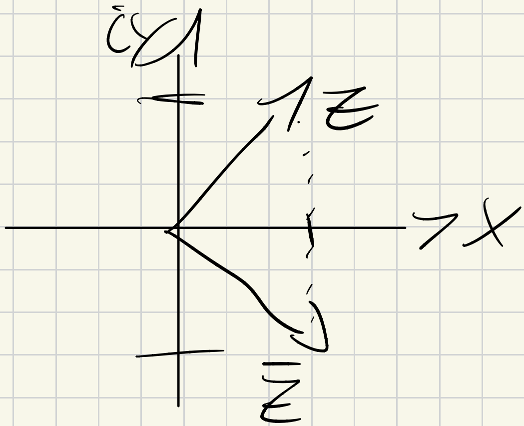
$\langle z, z \rangle \geq 0$

Euklidische Länge

$|z| := \sqrt{\langle z, z \rangle} = \sqrt{\operatorname{Re}(z\bar{z})}$

$\sqrt{\operatorname{Re}((x+iy)\cdot(x-iy))} = \sqrt{\operatorname{Re}(x^2+y^2)}$

$\sqrt{x^2+y^2} \qquad |z| = |\bar{z}|$



$$z^{-1} = \frac{\bar{z}}{|z|^2}$$

$$z\bar{z} = |z|^2$$

$$\frac{\cancel{z}\bar{z}}{|z|^2} = 1 \quad \Big| \, z^{-1}$$

$$z^{-1} = \frac{\bar{z}}{|z|^2}$$

$w = u + iv, \; z = x + iy$

$|wz| = |w| \cdot |z| \qquad \overline{wz} = \bar{w}\,\bar{z}$

$|wz| = \sqrt{\underbrace{wz\overline{wz}}} = \sqrt{w\bar{w}z\bar{z}}$

$= \sqrt{w\bar{w}}\sqrt{z\bar{z}} = |w| \cdot |z|$

$u, v, x, y \in \mathbb{R}$

$$\underbrace{(u^2+v^2)}_{|w|^2}\underbrace{(x^2+y^2)}_{|z|^2} = \underbrace{(ux-vy)^2+(uy+vx)^2}_{|wz|^2}$$

$\left|\frac{w}{z}\right| = \frac{|w|}{|z|} \qquad z \in \mathbb{C}^{\times}$

$$S^1 := \{ z \in \mathbb{C} \mid |z| = 1 \}$$

$c, |c| = 1, \quad z, |z| = 1$

$|c \cdot z| = |c| \cdot |z| = 1$

$$c = a + ib$$

$$\xi := \sqrt{\underbrace{\tfrac{1}{2}(|c| + a)}} + i\eta\sqrt{\underbrace{\tfrac{1}{2}(|c| - a)}}$$

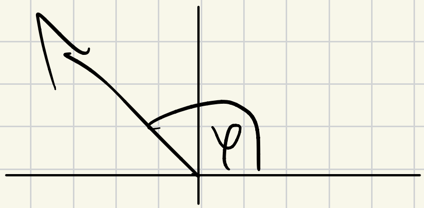
$$\underline{z^2 + 2cz + d} = 0$$

$$= (z + c)^2 + \underline{d - c^2}$$

$$z_{1/2} = -c \pm \sqrt{c^2 - d}$$

$2cz = q$, $p = 1$ $\Rightarrow$ pq-Formel

$z = r \cdot (\cos \varphi + i \sin \varphi)$, $r = |z|$



$z = |z| e^{i\varphi}$

$e^{i\varphi} = \cos \varphi + i \sin \varphi$

$$\cos \varphi = \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{(2\nu)!} \varphi^{2\nu}$$

$$\sin \varphi = \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{(2\nu+1)!} \cdot \varphi^{2\nu+1}$$

$$e^{i\varphi} = \sum_{\nu=0}^{\infty} \frac{i^\nu \cdot \varphi^\nu}{\nu!} \qquad i^{2\nu} = (i^2)^\nu = (-1)^\nu$$

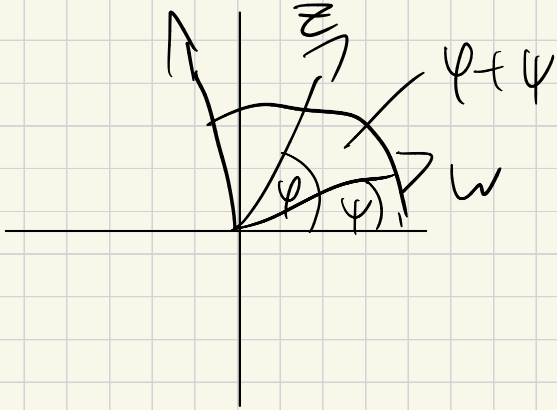
$$= \sum_{\nu=0}^{\infty} \frac{i^{2\nu} \cdot \varphi^{2\nu}}{(2\nu)!} + \frac{i^{2\nu+1} \cdot \varphi^{2\nu+1}}{(2\nu+1)!}$$

$$= \sum \underbrace{\frac{(-1)^\nu}{(2\nu)!} \cdot \varphi^{2\nu}}_{} + i \underbrace{\frac{(-1)^\nu}{(2\nu+1)!} \cdot \varphi^{2\nu+1}}_{}$$

$$\Rightarrow e^{i\varphi} = \cos\varphi + i \sin\varphi \quad \square$$

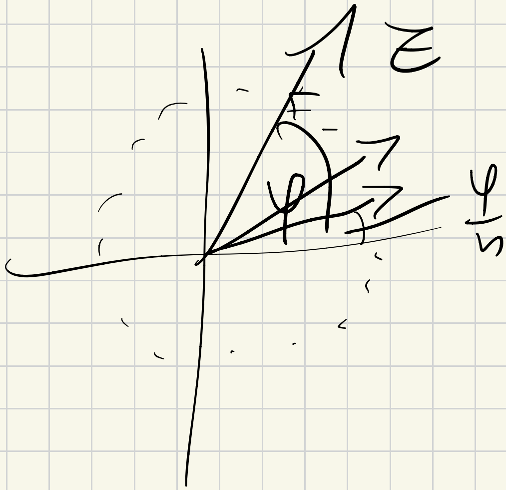
$w = |w| \cdot e^{i\psi}$, $z = |z| e^{i\varphi}$

$$w \cdot z = |w| \cdot |z| \, e^{i(\psi + \varphi)}$$



$z = |z| \cdot e^{i\varphi}$

$\sqrt[n]{|z|} \cdot e^{i\left(\frac{\varphi}{n}\right)}$



$\sqrt[n]{|z|} \cdot e^{i\left(\frac{\varphi + 2\pi \cdot a}{n}\right)} \quad , a = 0, \ldots, n-1$

$e^{i\left(\frac{\varphi + 2\pi \cdot n}{n}\right)} = e^{i\left(\frac{\varphi}{n} + 2\pi\right)}$