UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Axiomatische Verzamelingentheorie

2005/2006; 2nd Semester
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Homework Set # 11 *Deadline:* May 4th, 2006

Exercise 29 (total of nine points).

Define a multiplication $\otimes$ on tosets as follows: Let $\mathbf{X} = \langle X, \leq_X \rangle$ and $\mathbf{Y} = \langle Y, \leq_Y \rangle$ be tosets. Then $\mathbf{X} \otimes \mathbf{Y}$ is defined as $\langle X \times Y, \leq_{\mathrm{Hlex}} \rangle$ where

$$\langle x, y \rangle \leq_{\mathrm{Hlex}} \langle x', y' \rangle \iff y \leq_Y y' \vee (y = y' \wedge x \leq_X x').$$

Why is this called the "Hebrew lexicographic ordering" (1 point)?
Let $\times$ be the product of tosets with the ordinary lexicographic ordering. Give an example of tosets $\mathbf{X}$ and $\mathbf{Y}$ such that $\mathbf{X} \otimes \mathbf{Y}$ is not orderisomorphic to $\mathbf{X} \times \mathbf{Y}$ (3 points).
Prove that for ordinals α and β, we have that $\alpha \cdot \beta = \mathrm{o.t.}(\alpha \otimes \beta)$ (5 points).

Exercise 30 (total of ten points).

Let $\mathrm{Fun}(X, Y)$ be the set of functions from X to Y. Show that if there is an injection from Y to Y^*, then there is an injection from $\mathrm{Fun}(X, Y)$ to $\mathrm{Fun}(X, Y^*)$ (2 points). Show that if there is an injection from X to X^*, then there is an injection from $\mathrm{Fun}(X, Y)$ to $\mathrm{Fun}(X^*, Y)$ (2 points).

Use the Axiom of Choice to prove that there is an injection from ω_1 into $\mathrm{Fun}(\omega, 2)$ (2 points). Use this to prove that there is a bijection between $\mathrm{Fun}(\omega, \omega_1)$ and $\mathrm{Fun}(\omega, 2)$ (4 points).