



# Axiomatische Verzamelingsentheorie

2005/2006; 2nd Semester  
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## Homework Set # 4

Deadline: March 9th, 2006

### Exercise 11 (total of sixteen points).

- (1) Assume the empty set axiom, the power set axiom and the axiom scheme of replacement. Prove that the pairing axiom holds (5 points).
- (2) Assume the empty set axiom and the axiom scheme of replacement. Prove that the axiom scheme of separation (“subset collection”) holds (5 points).
- (3) Assume the axiom of infinity and the axiom scheme of replacement. Prove that the pairing axiom holds (6 points).

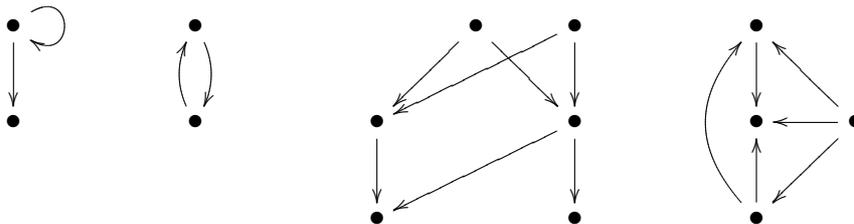
### Exercise 12 (total of eleven points).

Consider the following four graphs as LAST-structures. Here we interpret the  $\bullet$  symbols as distinct elements of the structure and the symbol  $\bullet \longrightarrow \circ$  as “ $\circ \in \bullet$ ”.

For each of the four structures, check whether

- (1) the empty set axiom (1/2 point),
- (2) the axiom of extensionality (1/2 point),
- (3) the pairing axiom (1/2 point), and
- (4) the axiom scheme of separation (“subset collection”, 1 point)

hold (in each case, give a brief argument; 2 1/2 points for each graph).



The fourth graph represents a well-known set. What is its usual name (1 point)?