Reasoning and Formal Modelling for Forensic Science
Lecture 10

Prof. Dr. Benedikt Löwe

2nd Semester 2010/11

A reminder from Lecture 6.

Adding a temporal dimension.

In many cases, our information changes over time. Further investigation of the situation reveals more values of 'Yes' and 'No', where previously we only had '?'. (Or, preferably not too often, reveals that some of our 'Yes' and 'No' values were false.)

We can see such a course of investigation as a sequence of partial situations where consistency changes values depending on what the current state of information is.

This is a first glimpse of how to include temporal information into the modelling (later in the course).

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We replaced the original relation DRIVE by two temporally distinct relations DRIVEACCIDENT and DRIVELAST.

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It is not only reflecting information change on the level of the investigators in the narrative (which would be formally represented by changes in the values of "Yes", "No" or "?" in the situations), but also reflecting an information change for the modeller who has to revise the set-up of the model.

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- So, in the actual work of the modeller, this is something to be taken care of.
- The disadvantage is that the situations S_1, S_2, S_3 and S_4 are not comparable anymore to S_5 since they are expressed in different formal languages.
- So, in a cleaned version of the model, we would need to go back to S_1 and change our formal language in order to reflect the additional information we received as modellers in later stages.

Bruner's Spiral Curriculum.

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Jerome Bruner (b. 1915)

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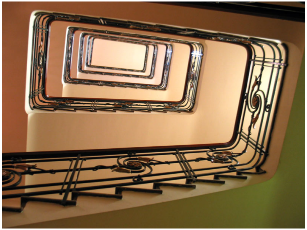
We learn by constantly revisiting (and possibly revising) past learning actions:



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The final version of the formal representation of a narrative should be phrased in one single language so that you can compare the controlled situations at the various stages.

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"online". If you want to design a system that can deal with changes that happen after you finalize the formal language, you want to have a very rich language that can react to new situations. You would **not** want to fix individuals, properties and relations in advance, because new individuals might show up during your work.

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"offline". If you have the entire story at your disposal, you can do *finite narrative modelling*: you read the entire narrative in advance and design a concrete and specific system that deals with all of the relevant aspects of the narrative.

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However, actually designing such a system is using a bit of both methodologies. While you go along the narrative and make modelling decisions to include elements in your system, you expand your language until you reach the end of the narrative.

In the process of *spiral modelling*, you then go back and re-assess the decisions you made earlier in order to get a homogeneous “offline” language and representation.

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Now we are able to express additional temporal information.

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We can introduce symbols for these: $@_i$, until_i, before_i, since_i, and $\mathrm{after}_\mathrm{i}$.

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- $\mathrm{since}_i\varphi$ is valid in S if φ is valid in S_j for all $j = i, ..., n$.
- $\mathrm{after}_i\varphi$ is valid in S if φ is valid in S_j for all $j = i + 1, ..., n$.
- $\mathrm{since}_i\varphi$ is invalid in S if φ is not valid in S_j for some $j = i, ..., n$.
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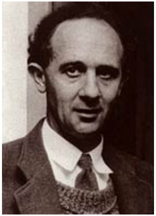

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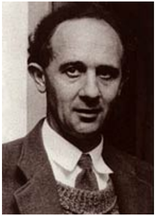

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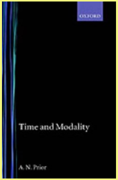

A. Prior, Time and Modality, Oxford University Press 1957

Hybrid Logic

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	STOLEN	KILLER
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u	*No*	*?*

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"In stage 5, something more complicated happens. Moore's new story about the driver switch after the accident forces us to change the setting of the modelling: we now need to have two relations 'driving the car at the time of the accident' and 'being the last driver of the car'." ... Also, we can now get rid of the individual u, since we know that this is about James.

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We use this information in our second round of the spiral to replace u by j throughout the stages, and by introducing the temporal component with the two moments t_{before} and t_{after}. With this information, we spiral back to S_1.

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j	No	No

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j	No	?	No

ϱ_0 $\exists y @_{\text{before}}\text{DRIVE}(y, c) \wedge \exists z @_{\text{after}}\text{DRIVE}(z, c)$.

ϱ_1 $@_{\text{before}}\text{STOLEN}(c) \rightarrow \neg @_{\text{before}}\text{DRIVE}(m, c)$.

ϱ_2 $\forall x @_{\text{before}}\text{DRIVE}(x, c) \rightarrow @_{\text{after}}\text{KILLER}(x)$.

Revisiting "Hit and Run" (4).

before	STOLEN	KILLER
m	No	No
c	?	No
j	No	No

after	STOLEN	KILLER
m	No	?
c	?	No
j	No	?

before

DRIVE	m	c	j
m	No	?	No
c	No	No	No
j	No	?	No

after

DRIVE	m	c	j
m	No	?	No
c	No	No	No
j	No	?	No

ϱ_0 $\exists y @_{\text{before}} \text{DRIVE}(y, c) \wedge \exists z @_{\text{after}} \text{DRIVE}(z, c)$.

ϱ_1 $@_{\text{before}} \text{STOLEN}(c) \rightarrow \neg @_{\text{before}} \text{DRIVE}(m, c)$.

ϱ_2 $\forall x @_{\text{before}} \text{DRIVE}(x, c) \rightarrow @_{\text{after}} \text{KILLER}(x)$.

Revisiting “Hit and Run” (4).

before	STOLEN	KILLER
m	No	No
c	?	No
j	No	No

after	STOLEN	KILLER
m	No	?
c	?	No
j	No	?

before

DRIVE	m	c	j
m	No	?	No
c	No	No	No
j	No	?	No

after

DRIVE	m	c	j
m	No	?	No
c	No	No	No
j	No	?	No

ϱ_0 $\exists y @_{\mathrm{before}} \mathrm{DRIVE}(y, c) \wedge \exists z @_{\mathrm{after}} \mathrm{DRIVE}(z, c)$.

ϱ_1 $@_{\mathrm{before}} \mathrm{STOLEN}(c) \rightarrow \neg @_{\mathrm{before}} \mathrm{DRIVE}(m, c)$.

ϱ_2 $\forall x @_{\mathrm{before}} \mathrm{DRIVE}(x, c) \rightarrow @_{\mathrm{after}} \mathrm{KILLER}(x)$.

These rules will be in place during the entire investigation and act as our consistency check.

Revisiting "Hit and Run" (4).

before	STOLEN	KILLER
m	No	No
c	No	No
j	No	No

after	STOLEN	KILLER
m	No	?
c	No	No
j	No	?

before

DRIVE	m	c	j
m	No	?	No
c	No	No	No
j	No	?	No

after

DRIVE	m	c	j
m	No	?	No
c	No	No	No
j	No	?	No

ϱ_0 $\exists y @_{\text{before}} \text{DRIVE}(y, c) \wedge \exists z @_{\text{after}} \text{DRIVE}(z, c)$.

ϱ_1 $@_{\text{before}} \text{STOLEN}(c) \to \neg @_{\text{before}} \text{DRIVE}(m, c)$.

ϱ_2 $\forall x @_{\text{before}} \text{DRIVE}(x, c) \to @_{\text{after}} \text{KILLER}(x)$.

These rules will be in place during the entire investigation and act as our consistency check.

When we move to stage 2, we change the values of STOLEN for both S_2^{before} and S_2^{after}.

Revisiting "Hit and Run" (4).

before	STOLEN	KILLER
m	No	No
c	No	No
j	No	No

after	STOLEN	KILLER
m	No	Yes
c	No	No
j	No	No

before

DRIVE	m	c	j
m	No	Yes	No
c	No	No	No
j	No	No	No

after

DRIVE	m	c	j
m	No	Yes	No
c	No	No	No
j	No	No	No

ϱ_0 $\exists y @_{\mathrm{before}} \mathrm{DRIVE}(y, c) \wedge \exists z @_{\mathrm{after}} \mathrm{DRIVE}(z, c)$.

ϱ_1 $@_{\mathrm{before}} \mathrm{STOLEN}(c) \rightarrow \neg @_{\mathrm{before}} \mathrm{DRIVE}(m, c)$.

ϱ_2 $\forall x @_{\mathrm{before}} \mathrm{DRIVE}(x, c) \rightarrow @_{\mathrm{after}} \mathrm{KILLER}(x)$.

These rules will be in place during the entire investigation and act as our consistency check.

When we move to stage 2, we change the values of STOLEN for both S_2^{before} and S_2^{after}.

Similarly, when we move to stage 3, we remove all of the question marks.

Revisiting "Hit and Run" (4).

before	STOLEN	KILLER
m	No	No
c	No	No
j	No	No

after	STOLEN	KILLER
m	No	?
c	No	No
j	No	?

before

DRIVE	m	c	j
m	No	?	No
c	No	No	No
j	No	?	No

after

DRIVE	m	c	j
m	No	?	No
c	No	No	No
j	No	?	No

ϱ_0 $\exists y @_{\text{before}} \text{DRIVE}(y, c) \wedge \exists z @_{\text{after}} \text{DRIVE}(z, c)$.

ϱ_1 $@_{\text{before}} \text{STOLEN}(c) \rightarrow \neg @_{\text{before}} \text{DRIVE}(m, c)$.

ϱ_2 $\forall x @_{\text{before}} \text{DRIVE}(x, c) \rightarrow @_{\text{after}} \text{KILLER}(x)$.

These rules will be in place during the entire investigation and act as our consistency check.

When we move to stage 2, we change the values of STOLEN for both S_2^{before} and S_2^{after}.

Similarly, when we move to stage 3, we remove all of the question marks. Stage 4 is just stage 2 again.

Revisiting "Hit and Run" (5).

before	STOLEN	KILLER
m	No	No
c	No	No
j	No	No

after	STOLEN	KILLER
m	No	?
c	No	No
j	No	?

before

DRIVE	m	c	j
m	No	?	No
c	No	No	No
j	No	?	No

after

DRIVE	m	c	j
m	No	?	No
c	No	No	No
j	No	?	No

ϱ_0 $\exists y @_{\text{before}} \text{DRIVE}(y, c) \wedge \exists z @_{\text{after}} \text{DRIVE}(z, c)$.

ϱ_1 $@_{\text{before}} \text{STOLEN}(c) \rightarrow \neg @_{\text{before}} \text{DRIVE}(m, c)$.

ϱ_2 $\forall x @_{\text{before}} \text{DRIVE}(x, c) \rightarrow @_{\text{after}} \text{KILLER}(x)$.

Revisiting "Hit and Run" (5).

before	STOLEN	KILLER
m	No	No
c	No	No
j	No	No

after	STOLEN	KILLER
m	No	?
c	No	No
j	No	?

before

DRIVE	m	c	j
m	No	?	No
c	No	No	No
j	No	?	No

after

DRIVE	m	c	j
m	No	No	No
c	No	No	No
j	No	Yes	No

ϱ_0 $\exists y @_{\text{before}} \text{DRIVE}(y, c) \wedge \exists z @_{\text{after}} \text{DRIVE}(z, c)$.

ϱ_1 $@_{\text{before}} \text{STOLEN}(c) \rightarrow \neg @_{\text{before}} \text{DRIVE}(m, c)$.

ϱ_2 $\forall x @_{\text{before}} \text{DRIVE}(x, c) \rightarrow @_{\text{after}} \text{KILLER}(x)$.

In stage 5, we finally use the temporal structure in a meaningful way. We learn change the after values of DRIVE. Still, rule ϱ_2 is consistent with Charles Moore being the killer.

Revisiting "Hit and Run" (5).

before	STOLEN	KILLER
m	No	No
c	No	No
j	No	No

after	STOLEN	KILLER
m	No	No
c	No	No
j	No	Yes

before

DRIVE	m	c	j
m	No	No	No
c	No	No	No
j	No	Yes	No

after

DRIVE	m	c	j
m	No	No	No
c	No	No	No
j	No	Yes	No

ϱ_0 $\exists y @_{\mathrm{before}} \mathrm{DRIVE}(y, c) \wedge \exists z @_{\mathrm{after}} \mathrm{DRIVE}(z, c)$.

ϱ_1 $@_{\mathrm{before}} \mathrm{STOLEN}(c) \rightarrow \neg @_{\mathrm{before}} \mathrm{DRIVE}(m, c)$.

ϱ_2 $\forall x @_{\mathrm{before}} \mathrm{DRIVE}(x, c) \rightarrow @_{\mathrm{after}} \mathrm{KILLER}(x)$.

In stage 5, we finally use the temporal structure in a meaningful way. We learn change the after values of DRIVE. Still, rule ϱ_2 is consistent with Charles Moore being the killer.

Finally, in stage 6, every question mark is resolved.

Another example.

Stage 1. *The police find Jean Bartington dead in her office with a knife in her back. The investigation of the crime scene shows that the murderer must have had a key to her office. There are only two people (except for Jean) who have a key: her secretary Paul and the building administrator Sheila. Paul was the person who found the body.*

Stage 2. *The forensic investigation finds fingerprints of Paul and Sheila on the knife.*

Stage 3. *The investigation shows that Paul's fingerprints resulted from him touching the knife when he found the body.*