Reasoning and Formal Modelling for Forensic Science
Lecture 4

Prof. Dr. Benedikt Löwe

2nd Semester 2010/11

Where are we right now?

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$$((p \wedge q \leftrightarrow r) \wedge \neg r) \rightarrow \neg p.$$

Proof by contradiction

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This formula contains two propositional variables (p and q), and the subformulas $\neg p$, $\neg q$, $p \wedge \neg p$ and $\neg q \rightarrow (p \wedge \neg p)$.

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E: Suppose all the women in Nigeria are married. Now there's a woman called Connie and she's not married. Can we say she lives in Nigeria or not?
S: What kind of clothes do they wear in Nigeria?
E: Just suppose the world is a strange one in which all the women in Nigeria are married.
S: We can say she's a Nigerian but she hasn't got married yet.

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quantifiers: "for all", "there is", "no"…

Quantifiers

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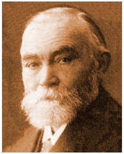
Historically, quantifiers entered logic very late:



Gottlob Frege (1848–1925)

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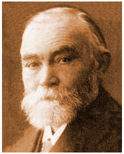

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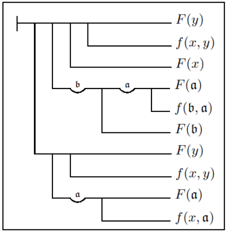

Theorem 71 from *Begriffsschrift*

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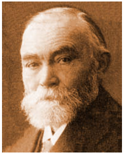

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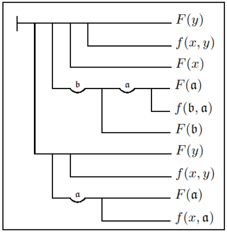

Theorem 71 from *Begriffsschrift*

Modern notation: $\forall x P(x)$ "for all x, $P(x)$ holds"; $\exists x P(x)$ "there is an x such that $P(x)$ holds.

Aristotelian syllogistics (1).

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Every animal is mortal.

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Every animal is mortal.
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The difference between syllogistics and full quantifier logic is that quantified statements are only allowed in very restricted argumentation contexts, governed by the rules of syllogistics.

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The difference between syllogistics and full quantifier logic is that quantified statements are only allowed in very restricted argumentation contexts, governed by the rules of syllogistics. In syllogistics, every argument is structurally of the above form: two quantified premisses and a quantified conclusion.

Aristotelian syllogistics (2).

Every man is an animal
Every animal is mortal

Every man is mortal

Aristotelian syllogistics (2).

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Every man is an animal Every animal is mortal	Every man is a donkey Every donkey is mortal	Every man is an animal
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Aristotelian syllogistics (2).

Every man is an animal	Every man is a donkey	Every man is an animal
Every animal is mortal	Every donkey is mortal	Every animal is made of stone
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Structurally, all of these arguments are the same:

$$\frac{\text{Every } A \text{ is } B \quad \text{Every } B \text{ is } C}{\text{Every } A \text{ is } C}$$

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Formal logic is the study of formal rules of argumentation, independent of content. The above form is called a "perfect syllogism": in its abstract form, its argument is compelling without any further argument.

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In syllogistics, we accept all three arguments above. Traditionally, these are called valid moods (from *modus*).

Aristotelian syllogistics (3).

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Every philosopher is mortal.
Some teacher is a philosopher.

Some teacher is mortal.

Aristotelian syllogistics (3).

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Every philosopher is mortal.
Some teacher is a philosopher.

Some teacher is mortal.

Every B is A.
Some C is B.

Some C is A.

Aristotelian syllogistics (3).

Every philosopher is mortal.
Some teacher is a philosopher.

Some teacher is mortal.

Every B is A.
Some C is B.

Some C is A.

Another valid mood!

Aristotelian syllogistics (4).

Every B is A.
Some C is B.

Some C is A.

Aristotelian syllogistics (4).

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This is not a valid mood, even though all of the sentences in our example are correct.

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Some teacher is a philosopher.

Every A is B.
Some C is B.

Some C is A.

This is not a valid mood, even though all of the sentences in our example are correct. It is not valid since there are possible interpretations of A, B, and C that make the inference invalid.

Aristotelian syllogistics (5).

Every A is B.
Some C is B.

Some C is A.

Aristotelian syllogistics (5).

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Every A is B.
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Example. A: "Dutch citizen", B "citizen of an EU country", C "Bulgarian citizen".

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Methodologically, not precise: how much do we know about dual citizens between Bulgaria and the Netherlands? To make this more precise, we define a controlled situation:

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Example. A: "Dutch citizen", B "citizen of an EU country", C "Bulgarian citizen".

Methodologically, not precise: how much do we know about dual citizens between Bulgaria and the Netherlands? To make this more precise, we define a controlled situation:

Suppose there are five people in a room: a, b, c, d, e. a is a Bulgarian citizen, b is a US citizen, c, d, and e are Dutch citizens. None of the five people has a dual nationality.

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We represent this by Venn diagrams.

Proving that a mood is invalid.

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Algorithm. Suppose you have an Aristotelian mood that you want to show invalid. The mood involves the terms A, B and C and has two premisses φ and ψ and a conclusion χ.

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Step 1. Draw the Venn diagram for the mood. This gives you an indication how to invalidate the mood.

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Step 3. Argue that each of the premisses φ and ψ is true in the controlled situation.

Step 4. Argue that the conclusion χ is not true in the controlled situation.

Proving that a mood is invalid: another example.

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There is an A that is B.
There is a B that is C.

Some A is a C.

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Proving that a mood is invalid: another example.

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Some A is a C.

Step 2. Take two individuals: a and b.

a is a male student. b is a female student. No one is both male and female.

Proving that a mood is invalid: another example.

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There is a B that is C.

Some A is a C.

Step 2. Take two individuals: a and b.

a is a male student. b is a female student. No one is both male and female.

A: male, B: student, C: female.

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Prof. Dr. Benedikt Löwe

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Suppose this mood is invalid. This means that there is a controlled situation with some individuals a_0, ..., a_n such that the properties A, B and C are defined for these individuals, and the premisses are true, but the conclusion is false.

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Fix the i from 3., then we have a_i which is both C and A. By 2., a_i must also be B. But then this is a contradiction to 1.

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No dead body that was killed less than 17 days ago, has *Calliphora stygia*.

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Called "enthymeme" by Aristotle (*Rhetorica*).

Psychology of reasoning and syllogisms (1).

Stanovich, K. E., & West, R. F. (1998). Individual differences in rational thought. *Journal of Experimental Psychology: General*, 127, 161-188

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All A are B.

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This is invalid (our male and female students example shows it).

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