

# Reasoning and Formal Modelling for Forensic Science Lecture 6

Prof. Dr. Benedikt Löwe

2nd Semester 2010/11

## Reminder: Controlled Situations with Relations.

A **controlled situation with relations** is a controlled situation together with some relations  $R_0, \dots, R_m$ .

We fix a controlled situation with relations  $S$ : collection  $E$  of individuals, some properties  $P_0, \dots, P_n$  and some relations  $R_0, \dots, R_m$ . We say

$P_i(e)$ is valid in $S$	if and only if $e$ has property $P_i$
$R_j(e, f)$ is valid in $S$	if and only if $e$ and $f$ are in relation $R_j$
$\varphi \wedge \psi$ is valid in $S$	if and only if $\varphi$ is valid in $S$ and $\psi$ is valid in $S$
$\varphi \vee \psi$ is valid in $S$	if and only if $\varphi$ is valid in $S$ or $\psi$ is valid in $S$
$\neg\varphi$ is valid in $S$	if and only if $\varphi$ is invalid in $S$
$\forall x\varphi$ is valid in $S$	if and only if no matter which $e \in E$ we choose, if we replace all occurrences of $x$ in $\varphi$ by $e$ , then this formula $\varphi_x^e$ is valid.
$\exists x\varphi$ is valid in $S$	if and only if there is some $e \in E$ such that if we replace all occurrences of $x$ in $\varphi$ by $e$ , then this formula $\varphi_x^e$ is valid.

## Important to note!

Note that the semantics of quantifier logic have a **mathematical** definition, and thus if you fix a controlled situation with relations, whether a given statement is valid in that situation or not is not debatable.

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Note that the semantics of quantifier logic have a **mathematical** definition, and thus if you fix a controlled situation with relations, whether a given statement is valid in that situation or not is not debatable.

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In our examples, we do not claim that our situations represent the **entire** situation. In fact, they do not. This will become important later.

# DQL: Example 1 (repeated from last week).

Police report, Colorado Springs, 15 Feb 2011, 2:55 pm:

A female victim called 911 to report that she had been stabbed near the Stargazers Theater ... The victim reported a Hispanic male in his late 20's to early 30's attempted to rob her, and he stabbed her in the stomach area. Officers and medical personnel contacted the victim in the south parking lot of the Stargazers Theatre and she was transported to the hospital to have the knife removed from her lower stomach area. The victim described the suspect as a Hispanic male in his late 20's to early 30's, approximately 5-10 in height with a heavier build and a ponytail. The suspect was reported to be wearing a plain black long sleeve shirt, jeans, and black gloves. Officers searched the area but were unable to locate the suspect.

Individuals:  $f$  (female),  $m$  (male),  $o$  (officers). Properties:  $H$  (hospitalized). Relations:  $S$  (stabbed),  $L$  (located).

	$H$	$S$	$f$	$m$	$o$	$L$	$f$	$m$	$o$
$f$	Yes	$f$	No	No	No	$f$	No	No	No
$m$	No	$m$	Yes	No	No	$m$	No	No	No
$o$	No	$o$	No	No	No	$o$	Yes	No	No

- ▶ Someone who stabbed someone else is still not located.

$$\exists x(\exists yS(x, y) \wedge \forall z\neg L(z, x))$$

$$S(m, f) \wedge (\neg L(f, m) \wedge \neg L(m, m) \wedge \neg L(o, m))$$

$\rightsquigarrow$  **YES!**

- ▶ There is someone who got stabbed but was not hospitalized.

$$\exists x(\exists yS(y, x) \wedge \neg H(x))$$

$$S(m, f) \wedge H(f), \neg\exists yS(y, m), \neg\exists yS(y, o)$$

$\rightsquigarrow$  **No!**

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*Police report, Colorado Springs, 14 Feb 2011, 1:30 am:*

*Officers were dispatched ... to investigate a shooting ... Officers contacted the victim and observed that he had been shot in the legs. Further investigation revealed that the victim was attempting to get into his vehicle in the parking lot, when a pick up truck pulled into the parking lot and began shooting at the victim. Officers contacted several witnesses to this incident; however, no witnesses were able to provide a suspect description. Officers received information that the suspect vehicle was a full size pick up truck, possibly silver or green in color. The victim was transported to a local hospital where he was treated for serious, but non life threatening injuries.*

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Individuals:  $v$  (victim),  $p$  (pick up truck). Properties:  $H$  (hospitalized).

Relations:  $S$  (shot).

	$H$	$S$	$v$	$p$
$v$	Yes	$v$	No	No
$p$	No	$p$	Yes	No

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- ▶ More interesting: there are properties not included in our situation, such as “silver” or “green” because we do not know which value the pick up truck has:

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$\exists x(S(x, v))$

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$\rightsquigarrow$  **YES!**

- ▶ More interesting: there are properties not included in our situation, such as “silver” or “green” because we do not know which value the pick up truck has: **uncertainty**.

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$v$	Yes	No	No	$v$	No	No
$p$	No	?	?	$p$	Yes	No

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	$H$	$I$	$G$	$S$	$v$	$p$
$v$	Yes	No	No	$v$	No	No
$p$	No	?	?	$p$	Yes	No

Partially controlled situations give rise to **consistency statements**: some statements are not true or false in a partially controlled situation, but **consistent** with it or **inconsistent** with it.

## Incomplete descriptions (2).

B. Robertson, G. A. Vignaux, Interpreting Evidence. Evaluating Forensic Science in the Courtroom. Wiley 1995:

*the word “consistent” ... is common use by forensic scientists, pathologists and lawyers. To a scientist, ... “consistent with” is simply the opposite of “inconsistent with”. The definition of “inconsistent” is precise and narrow. Two events are inconsistent with one another if they cannot possibly occur together. ... Unfortunately for clear communication, [researchers] found that lawyers usually interpret “consistent with” as meaning “reasonably strongly supporting”, while scientists use it in its strict logical and neutral meaning. (p. 56)*

# Partially Controlled Situations (1).

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A **partially controlled situation (with relations)** is a collection of individuals, some properties (and relations), together with **partial** assignments of values to the properties and relations, i.e., a table that has entries 'Yes', 'No', and '?'.

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The semantics for partially controlled situations is exactly like that for controlled situations, with the exception that not every truth value is determined and therefore, we need to make a distinction between "invalid" and "not valid".

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In partially controlled situations, some statements are valid, some are invalid, and some are undetermined.

## Partially Controlled Situations (2).

Fix a partially controlled situation with relations  $S$ , i.e., a collection  $E$  of individuals, some properties  $P_0, \dots, P_n$  and some relations  $R_0, \dots, R_m$  with possible values 'Yes', 'No', and '?'. We say

$P_i(e)$ is valid in $S$	if and only if $e$ has property $P_i$
$P_i(e)$ is invalid in $S$	if and only if $e$ does not have property $P_i$
$R_j(e, f)$ is valid in $S$	if and only if $e$ and $f$ are in relation $R_j$
$R_j(e, f)$ is invalid in $S$	if and only if $e$ and $f$ are not in relation $R_j$
$\varphi \wedge \psi$ is invalid in $S$	if and only if $\varphi$ is invalid in $S$ or $\psi$ is invalid in $S$
$\neg\varphi$ is valid in $S$	if and only if $\varphi$ is invalid in $S$
$\neg\varphi$ is invalid in $S$	if and only if $\varphi$ is valid in $S$
$\forall x\varphi$ is invalid in $S$	if and only if there is an $e \in E$ such that, if we replace all occurrences of $x$ in $\varphi$ by $e$ , then this formula $\varphi_x^e$ is invalid.

The other definitions of validity ( $\varphi \wedge \psi$ ,  $\forall x\phi$ ,  $\exists x\phi$ ) stay the same.

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Fix a partially controlled situation with relations  $S$  and a finite list of formulas  $\varrho = \{\varrho_0, \dots, \varrho_k\}$ . The formulas in this list are called **rules**. Assume that some value, either of a property or a relation, is undecided, i.e., has value “?”. Let  $S_{\text{Yes}}$  be the extension of  $S$  where this value is replaced by “Yes”, and  $S_{\text{No}}$  be the extension where this value is replaced by “No”.

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We say that the answer “Yes” (“No”) is **inconsistent with  $S$  and  $\varrho$**  if there is a rule  $\varrho_i$  such that  $\varrho_i$  is invalid in  $S_{\text{Yes}}$  ( $S_{\text{No}}$ ).

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We say that the answer “Yes” (“No”) is **inconsistent with  $S$  and  $\varrho$**  if there is a rule  $\varrho_i$  such that  $\varrho_i$  is invalid in  $S_{\text{Yes}}$  ( $S_{\text{No}}$ ).

A formula  $\varphi$  is called **consistent with  $S$  and  $\varrho$**  if it is not inconsistent with  $S$  and  $\varrho$ .

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*An old lady is found dead at the bottom of her staircase by her grandson John. The neighbours heard a loud fight the evening before, but no one checked. John's mother reports to the police that John and his grandmother had been quarreling for weeks about a car that she had promised John for his 18th birthday. It is uncertain whether John was in the house the evening of the death. There are no signs of breaking into the house. The autopsy reveals that the cause of death was blunt trauma, but it is impossible to say whether this was a fall down the stairs or a hit on the head.*

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We model this situation by using three individuals:  $j$  (John),  $\ell$  (the old lady),  $u$  (an unknown person). We use properties: Dead, Blunt, Fall, and Accident, standing for “is dead”, “has blunt trauma”, “fell down the stairs”, and “had an accident without influence of others”.

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	Dead	Blunt	Fall	Accident
$j$	No	No	No	?
$\ell$	Yes	Yes	?	?
$u$	?	?	?	?

## Consistency & inconsistency (3).

	Dead	Blunt	Fall	Accident
<i>j</i>	No	No	No	?
<i>ℓ</i>	Yes	Yes	?	?
<i>u</i>	?	?	?	?

We add relations  $\text{Hit}(x, y)$  and  $\text{Push}(x, y)$  for “ $x$  hit  $y$ ” and “ $x$  pushed  $y$  down the stairs”.

Hit	<i>j</i>	<i>ℓ</i>	<i>u</i>	Push	<i>j</i>	<i>ℓ</i>	<i>u</i>
<i>j</i>	No	?	?	<i>j</i>	No	?	?
<i>ℓ</i>	No	No	?	<i>ℓ</i>	No	No	?
<i>u</i>	?	?	No	<i>u</i>	?	?	No

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	Dead	Blunt	Fall	Accident
<i>j</i>	No	No	No	?
<i>ℓ</i>	Yes	Yes	?	?
<i>u</i>	?	?	?	?

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Hit	<i>j</i>	<i>ℓ</i>	<i>u</i>	Push	<i>j</i>	<i>ℓ</i>	<i>u</i>
<i>j</i>	No	?	?	<i>j</i>	No	?	?
<i>ℓ</i>	No	No	?	<i>ℓ</i>	No	No	?
<i>u</i>	?	?	No	<i>u</i>	?	?	No

And **rules** that reflect the logical or physical connections between the properties or relations:

## Consistency & inconsistency (3).

	Dead	Blunt	Fall	Accident
<i>j</i>	No	No	No	?
<i>ℓ</i>	Yes	Yes	?	?
<i>u</i>	?	?	?	?

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Hit	<i>j</i>	<i>ℓ</i>	<i>u</i>	Push	<i>j</i>	<i>ℓ</i>	<i>u</i>
<i>j</i>	No	?	?	<i>j</i>	No	?	?
<i>ℓ</i>	No	No	?	<i>ℓ</i>	No	No	?
<i>u</i>	?	?	No	<i>u</i>	?	?	No

And **rules** that reflect the logical or physical connections between the properties or relations:

- ▶  $\varrho_0: \text{Dead}(x) \leftrightarrow \text{Blunt}(x)$
- ▶  $\varrho_1: \text{Blunt}(x) \leftrightarrow \text{Fall}(x) \vee \exists y \text{Hit}(y, x)$
- ▶  $\varrho_2: \text{Fall}(x) \leftrightarrow \text{Accident}(x) \vee \exists y \text{Push}(y, x)$

# Consistency & inconsistency (4).

	Dead	Blunt	Fall	Accident
<i>j</i>	No	No	No	?
<i>ℓ</i>	Yes	Yes	?	?
<i>u</i>	?	?	?	?

  

Hit	<i>j</i>	<i>ℓ</i>	<i>u</i>	Push	<i>j</i>	<i>ℓ</i>	<i>u</i>
<i>j</i>	No	?	?	<i>j</i>	No	?	?
<i>ℓ</i>	No	No	?	<i>ℓ</i>	No	No	?
<i>u</i>	?	?	No	<i>u</i>	?	?	No

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## Consistency &amp; inconsistency (4).

	Dead	Blunt	Fall	Accident
<i>j</i>	No	No	No	?
<i>ℓ</i>	Yes	Yes	?	?
<i>u</i>	?	?	?	?

  

Hit	<i>j</i>	<i>ℓ</i>	<i>u</i>	Push	<i>j</i>	<i>ℓ</i>	<i>u</i>
<i>j</i>	No	?	?	<i>j</i>	No	?	?
<i>ℓ</i>	No	No	?	<i>ℓ</i>	No	No	?
<i>u</i>	?	?	No	<i>u</i>	?	?	No

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- ▶  $\varrho_2: \text{Fall}(x) \leftrightarrow \text{Accident}(x) \vee \exists y \text{Push}(y, x)$

Consider  $\varrho_1$  for  $x = \ell$ .

$$\text{Blunt}(\ell) \leftrightarrow \text{Fall}(\ell) \vee \exists y \text{Hit}(y, \ell)$$

# Consistency & inconsistency (4).

	Dead	Blunt	Fall	Accident
<i>j</i>	No	No	No	?
<i>ℓ</i>	Yes	Yes	?	?
<i>u</i>	?	?	?	?

  

Hit	<i>j</i>	<i>ℓ</i>	<i>u</i>	Push	<i>j</i>	<i>ℓ</i>	<i>u</i>
<i>j</i>	No	?	?	<i>j</i>	No	?	?
<i>ℓ</i>	No	No	?	<i>ℓ</i>	No	No	?
<i>u</i>	?	?	No	<i>u</i>	?	?	No

- ▶  $\varrho_0: \text{Dead}(x) \leftrightarrow \text{Blunt}(x)$
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## Consistency &amp; inconsistency (4).

	Dead	Blunt	Fall	Accident
<i>j</i>	No	No	No	?
<i>ℓ</i>	Yes	Yes	?	?
<i>u</i>	?	?	?	?

  

Hit	<i>j</i>	<i>ℓ</i>	<i>u</i>	Push	<i>j</i>	<i>ℓ</i>	<i>u</i>
<i>j</i>	No	?	?	<i>j</i>	No	?	?
<i>ℓ</i>	No	No	?	<i>ℓ</i>	No	No	?
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<i>j</i>	No	?	?	<i>j</i>	No	?	?
<i>ℓ</i>	No	No	?	<i>ℓ</i>	No	No	?
<i>u</i>	?	?	No	<i>u</i>	?	?	No

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Hit						Push		
	<i>j</i>	No	?	?		<i>j</i>	No	?
	<i>ℓ</i>	No	No	?		<i>ℓ</i>	No	No
	<i>u</i>	?	?	No		<i>u</i>	?	?
			No				No	

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<i>j</i>	No	?	?	<i>j</i>	No	?	?
<i>ℓ</i>	No	No	?	<i>ℓ</i>	No	No	?
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No: our relations  $R$  were **binary**, i.e., they always relate one individual to another, like “taller than”, “killed”, “shot”.

In general, some relations are more complicated:

“The suspect shot the victim with a .5 caliber rifle”

“The police found a .23 caliber gun on the suspect”

**Ternary** relations:

- ▶  $S(s, v, r) \rightsquigarrow$  “ $s$  shot  $v$  with  $r$ ”.
- ▶  $F(p, g, s) \rightsquigarrow$  “ $p$  found  $g$  on  $s$ ”

With ternary relations (and relations with more entries), we can generalize our semantics to “**full quantifier logic**”.

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Once you have transformed a description of a scenario into mathematics, everything just becomes following an algorithm and applying the definitions correctly.

The difficult step is the **link** between the scenario (given to you in natural language or –even worse– by personal experience) and the mathematical representation.

If someone gives me a police report, how do I come up with the **right** individuals, properties, relations, and rules in order to do the formal assessment?

## Modelling (2).

**Modelling gone wrong:** in the scenario with the old lady, suppose that we formalized badly by a controlled situation  $S^{\text{bad}}$  where we picked only two individuals  $j$  and  $\ell$ , only three properties, Dead, Blunt, and Fall, and the two relations Hit( $x, y$ ) and Push( $x, y$ ) with

		Dead	Blunt	Fall	
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	$j$	No	$j$	No	?
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- ▶ “John is innocent” means that both  $\text{Push}(j, \ell)$  and  $\text{Hit}(j, \ell)$  get the answer “No”.
- ▶ But if  $\text{Push}(j, \ell)$  gets the answer “No”, then  $\text{Fall}(\ell)$  gets the answer “No” by  $\varrho_2$ .

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- ▶ “John is innocent” means that both  $\text{Push}(j, \ell)$  and  $\text{Hit}(j, \ell)$  get the answer “No”.
- ▶ But if  $\text{Push}(j, \ell)$  gets the answer “No”, then  $\text{Fall}(\ell)$  gets the answer “No” by  $\varrho_2$ .
- ▶ And then  $\text{Hit}(j, \ell)$  must get the answer “Yes” by rule  $\varrho_1$ .
- ▶ So, both  $\text{Push}(j, \ell)$  and  $\text{Hit}(j, \ell)$  getting the answer “No” would violate either rule  $\varrho_1$  or  $\varrho_2$ , and is thus inconsistent with  $S^{\text{bad}}$  and  $\varrho$ .

# Modelling (4).

## **Important.**

Our modelling process has a profound influence on our assessment of consistency and inconsistency. It is easily possible to change an assessment of “some assumption is consistent” to “some assumption is inconsistent” by choosing differently in the modelling process. Therefore we need to be extremely careful in the modelling process.

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The fight and the information about the promised car are not part of our formalisation, as they do not allow any formal conclusions. They have an influence on the assessment of the situation on a very different level: if there are several **consistent** scenarios (accident, John pushed, unknown person pushed, John hit, unknown person hit), we need to assess how likely they are. In order to do that, we use additional information, such as motivation for actions and circumstances.

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It would be possible to add these to the formalization as well.

# Syntax, Semantics, Pragmatics

Reasoning and  
Formal Modelling  
for Forensic  
Science  
Lecture 6

Prof. Dr. Benedikt  
Löwe

- ▶ **Syntax.** The list of individuals, properties, and relations.

# Syntax, Semantics, Pragmatics

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- ▶ **Semantics.** The assignment of “Yes”, “No”, and “?” to the properties and relations.
- ▶ **Pragmatics.** The interpretation of the model:
  - ▶ In the case of two consistent assumptions, assessment of the likelihood that they are true.
  - ▶ Interpretation of natural language statements in the model.

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We can see such a course of investigation as a **sequence** of partial situations where consistency changes values depending on what the current state of information is.

This is a first glimpse of how to include temporal information into the modelling (later in the course).

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*The police are investigating a disturbing hit and run death of a young girl. The autopsy reveals a partial license plate number visible on the girl's body in a bruise. A hit on the partial license plate number brings the police to the home of Charles Moore, a gentlemanly seventy-three-year-old. He claims his car had been stolen.*

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*However, a search reveals that the car is in the garage.*

*Moore confesses that he was behind the wheel. He spotted the girl in the middle of the road, went to brake, and instead stepped on the accelerator.*

*The police examine Moore's car. They notice that the driver's seat is pushed too close for his height and the car radio is set to blast a hip-hop station.*

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*The police are investigating a disturbing hit and run death of a young girl. The autopsy reveals a partial license plate number visible on the girl's body in a bruise. A hit on the partial license plate number brings the police to the home of Charles Moore, a gentlemanly seventy-three-year-old. He claims his car had been stolen.*

*However, a search reveals that the car is in the garage.*

*Moore confesses that he was behind the wheel. He spotted the girl in the middle of the road, went to brake, and instead stepped on the accelerator.*

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*They ask Moore if anyone else drives his car. Moore admits that after hitting the girl, he'd banged his head. His grandson James drove him home.*

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*Taking a closer look at Charles Moore's car, an investigator retrieves a small piece of tooth embedded in the steering wheel matching James Moore.*