Universiteit van Amsterdam
Institute for Logic, Language and Computation

Reasoning and Formal Modelling for Forensic Science

2010/2011; 2nd Semester

Prof. Dr. Benedikt Löwe

Semantics for partially controlled situations and the "Hit and Run" example.

These notes contain additional information concerning the semantics for *partially controlled situations* (Lecture 6) and go through the example "Hit and Run" that we did on the blackboard in all detail.

A *partially controlled situation with relations* consists of a set $E = \{e_0, ..., e_k\}$ of individuals, some properties $P_0, ..., P_n$ and some relations $R_0, ..., R_m$. For each property P_i and each individual e_j, we either say "e_j has property P_i" (abbreviation: $P_i(e_j)$), "e_j does not have property P_i, or "it is unknown whether e_j has property P_i". Similarly, for any relation R_i and any individuals e_j and $e_{j'}$, we either say "e_j and $e_{j'}$ are in relation R_i" (abbreviation: $R_i(e_j, e_{j'})$), "e_j and $e_{j'}$ are not in relation R_i" or "it is unknown whether e_j and $e_{j'}$ are in relation R_i".
We fix these statements in tables with the values "Yes", "No", and "?". A partially controlled situation S consists of assignments of these three values for every property and relation. The following is a complete description of the meaning of the notions "valid" and "invalid" in a given partially controlled situation:

(1) $P_i(e)$ is valid in S if and only if e has property P_i.
(2) $P_i(e)$ is invalid in S if and only if e does not have property P_i.
(3) $R_j(e, f)$ is valid in S if and only if e and f are in relation R_j.
(4) $R_j(e, f)$ is invalid in S if and only if e and f are not in relation R_j.
(5) $\varphi \wedge \psi$ is valid in S if and only if φ is valid in S and ψ is valid in S.
(6) $\varphi \wedge \psi$ is invalid in S if and only if φ is invalid in S or ψ is invalid in S.
(7) $\varphi \vee \psi$ is valid in S if and only if φ is valid in S or ψ is valid in S.
(8) $\varphi \vee \psi$ is invalid in S if and only if φ is invalid in S and ψ is valid in S.
(9) $\varphi \rightarrow \psi$ is valid in S if and only if φ is invalid in S or ψ is valid in S.
(10) $\varphi \rightarrow \psi$ is invalid in S if and only if φ is valid in S and ψ is invalid in S.
(11) $\neg\varphi$ is valid in S if and only if φ is invalid in S.
(12) $\neg\varphi$ is invalid in S if and only if φ is valid in S.
(13) $\forall x\varphi$ is valid in S if and only if no matter which $e \in E$ we choose, if we replace all occurrances of x in φ by e, then this formula (denoted by $\varphi\frac{e}{x}$) is valid.
(14) $\forall x\varphi$ is invalid in S if and only if there is an $e \in E$ such that, if we replace all occurrances of x in φ by e, then this formula (denoted by $\varphi\frac{e}{x}$) is invalid.
(15) $\exists x\varphi$ is valid in S if and only there is some $e \in E$ such that if we replace all occurrances of x in φ
by e, then this formula (denoted by $\varphi\frac{e}{x}$) is valid.
(16) $\exists x\varphi$ is invalid in S if and only if no matter which $e \in E$ we choose, if we replace all occurrances of x in φ by e, then this formula (denoted by $\varphi\frac{e}{x}$) is invalid.

In the following, we formalize an investigation as described in the story "Hit and Run" (see Lecture 6) in six steps, building a snapshot of the current information situation at six particular times of the investigation. (**Important.** Keep in mind that such a formalization is not unique: it is always a choice of the modeller to include a particular individual or property or relation!)

Stage 1 (situation S_1). *The police are investigating a disturbing hit and run death of a young girl. The autopsy reveals a partial license plate number visible on the girl's body in a bruise. A hit* *on the partial license plate number brings the police to the home of Charles Moore, a gentlemanly seventy-three-year-old. He claims his car had been stolen.*

Stage 2 (situation S_2). *However, a search reveals that the car is in the garage.*

Stage 3 (situation S_3). *Moore confesses that he was behind the wheel. He spotted the girl in the middle of the road, went to brake, and instead stepped on the accelerator.*

Stage 4 (situation S_4). *The police examine Moore's car. They notice that the driver's seat is pushed too close for his height and the car radio is set to blast a hip-hop station.*

Stage 5 (situation S_5). *They ask Moore if anyone else drives his car. Moore admits that after hitting the girl, he'd banged his head. His grandson James drove him home.*

Stage 6 (situation S_6). *Taking a closer look at Charles Moore's car, an investigator retrieves a small piece of tooth embedded in the steering wheel matching James Moore.*

Situation S_1 consists of the individuals m (Charles Moore), c (the car), and u (an unknown driver). We include the unknown driver in order to be able to express that someone else drove Moore's car. We use the properties STOLEN and KILLER and the relation DRIVE, standing for "was stolen", "is the killer of the girl", and "was driving at the time of the accident". The semantics of this partially controlled situation is given by:

	STOLEN	KILLER
m	No	?
c	?	No
u	No	?

DRIVE	m	c	u
m	No	?	No
c	No	No	No
u	No	?	No

We add the rules $\varrho = \{\varrho_0, \varrho_1, \varrho_2\}$:

ϱ_0 $\exists y \text{DRIVE}(y, c)$.
ϱ_1 $\text{STOLEN}(c) \to \neg\text{DRIVE}(m, c)$.
ϱ_2 $\forall x \text{DRIVE}(x, c) \to \text{KILLER}(x)$.

In situation S_1, we can now analyse what is consistent with S_1 and ϱ. Remember what it means for an assumption to be consistent with S_1 and ϱ: not to be inconsistent, i.e., in the modified situation where the assumption is true, none of the rules is invalid.

Proposition 1. *The assumption "Moore is not the killer" is consistent with S_1 and ϱ.*

Proof. The modified situation S_1^* would be exactly as S_1 with the property table

	STOLEN	KILLER
m	No	No
c	?	No
u	No	?

Let us check that none of the three rules is invalid in S_1^*:

- Rule ϱ_0 says $\exists y \text{DRIVE}(y, c)$. We consider our semantics (line (16) above): for an existential formula to be invalid, all instances must be invalid. We consider the three instances: $\text{DRIVE}(m, c)$ is neither valid or invalid; $\text{DRIVE}(c, c)$ is invalid; $\text{DRIVE}(u, c)$ is neither valid nor invalid. So, there are some instances that are not invalid, and therefore ϱ_0 is not invalid.
- Rule ϱ_1 says $\text{STOLEN}(c) \to \neg\text{DRIVE}(m, c)$. We consider our semantics (line (10) above). For an implication to be invalid, the antecedent must be valid and the conclusion invalid. But the antecedent is neither valid nor invalid.
- Rule ϱ_2 says $\forall x \text{DRIVE}(x, c) \to \text{KILLER}(x)$. Once more, we consider our semantics (this time line (14)) and see that there must be an invalid instance for a universally quantified formula to be invalid. So, we need to check the three instances: $\text{DRIVE}(m, c) \to \text{KILLER}(m)$, $\text{DRIVE}(c, c) \to \text{KILLER}(c)$, and $\text{DRIVE}(u, c) \to \text{KILLER}(u)$. Each of these is an implication, so we can use line (10) once more and know that we only have to check whether the antecedent

is valid and the conclusion is invalid. In the first and third instance, the antecedent is neither valid nor invalid; in the second case, the antecedent is invalid.

□

Similar arguments show that "Moore is the killer", "Moore didn't drive at the time of the accident", "Moore drove at the time of the accident", "the car was stolen", "the car wasn't stolen" are consistent with S_1 and ϱ.

At stage 2 of the story, we learn that the car was not stolen, so we modify the properties in S_1 to get

	STOLEN	KILLER
m	No	?
c	No	No
u	No	?

and obtain a new partially controlled situation S_2. We can easily check that all mentioned assumptions consistent with S_1 remain consistent with S_2, except for "the car was stolen" and "the car wasn't stolen" (since STOLEN(c) now has a fixed truth value).

At stage 3 of the story, Moore confesses and gives truth values for all of the remaining ? signs in the tables: he drove the car, he killed the girl and there was no "unknown person". With the tables

	STOLEN	KILLER
m	No	Yes
c	?	No
u	No	No

DRIVE	m	c	u
m	No	Yes	No
c	No	No	No
u	No	No	No

we obtain a **controlled situation** (not a partially controlled situation), and so questions about consistency and inconsistency don't make sense anymore.

Stage 4 of the story casts doubt on the confession and thus reverts to the situation of uncertainty before the confession ($S_4 = S_2$).

In stage 5, something more complicated happens. Moore's new story about the driver switch after the accident forces us to change the setting of the modelling: we now need to have two relations "driving the car at the time of the accident" and "being the last driver of the car". The police have already determined that Moore was not the last driver of the car. Also, we can now get rid of the individual u, since we know that this is about James.
The new situation S_5 has individuals m (Moore), c (car), and j (James), properties STOLEN and KILLER and the relations DRIVEACCIDENT and DRIVELAST. The semantics of this partially controlled situation S_5 is given by:

	STOLEN	KILLER
m	No	?
c	No	No
j	No	?

DRIVEACCIDENT	m	c	j
m	No	?	No
c	No	No	No
j	No	?	No

DRIVELAST	m	c	j
m	No	No	No
c	No	No	No
j	No	Yes	No

We need to modify the rules slightly and get $\varrho^* = \{\varrho_0^*, \varrho_2^*\}$:

ϱ_0^* $\exists y$DRIVEACCIDENT(y, c).
ϱ_2^* $\forall x$DRIVEACCIDENT$(x, c) \to$ KILLER(x).

We check (as above) that "Moore was the killer" and "James was the killer" are both consistent with S_5.

Finally, the police find out that James was the driver at the time of the accident and thus (by rule ϱ_2^*) the killer. This new information removes the last four question marks and with the definitions

	STOLEN	KILLER
m	No	No
c	No	No
j	No	Yes

DRIVEACCIDENT	m	c	j
m	No	No	No
c	No	No	No
j	No	Yes	No

DRIVELAST	m	c	j
m	No	No	No
c	No	No	No
j	No	Yes	No

we obtain the final situation S_6 which again is a **controlled situation** (i.e., not partial).