

HOMWORK SET #2

Capita Selecta: Set Theory
2016/17: 1st Semester; block a
Universiteit van Amsterdam

Homework. There will be six homework sheets; handed in by each student individually. Homework has to contain the name and student ID of the student. If your homework is handwritten, make sure that it is legible. You submit your solutions either by e-mail to `h(dot)nobrega(at)uva(dot)nl` or in person before the Tuesday lecture or by placing them in Hugo's mailbox at the ILLC at Science Park 107.

Deadline. The first homework is due on **Tuesday, 20 September 2016** before the lecture.

In all of the exercises, work in a sufficiently strong metatheory.

1. Consider the class of ordinals Ord and check which axioms of ZFC hold in the structure (Ord, \in) .
2. Show that in $\mathbf{V}_{\omega+\omega}$, it is not true that every well-ordering is isomorphic to an ordinal. (*Hint.* Find a wellordering isomorphic to $\omega + \omega$ within $\mathbf{V}_{\omega+\omega}$.)
3. Let $\Phi(x, y)$ be the formula expressing “ x is a binary well-founded relation on y ”. Show that Φ is absolute for transitive models of $\text{ZF}-$.
4. Let S be any set of \mathcal{L}_\in -sentences. We say that a formula φ is *S-provably* Δ_1 if there is a Σ_1 -formula σ and a Π_1 -formula π such that $S \vdash \varphi \leftrightarrow \sigma \leftrightarrow \pi$. Show that if φ is *S-provably* Δ_1 , then φ is absolute between transitive models of S .