Oracle inequalities for the matrix completion problem

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Abstract

We show that empirical risk minimization procedures (ERM) and penalized empirical risk minimization procedures (PERM) satisfy oracle inequalities in a general framework. We underline a fundamental difference between the general exact oracle inequalities obtained in [7, 2, 1] and the non-exact oracle inequalities that we proved.

In the matrix completion setup where one observe n i.i.d. output/input couples $(Y_i, X_i)_{i=1}^n$ where $Y_i \in \mathbb{R}$ and $X_i \in \mathbb{R}^{m\times T}$ (and usually the dimension of the input variable is much larger than the number of observations: $mT >> n$), we apply these results to show that PERM procedures using the nuclear norm as a penalty function (cf. [3, 8]) satisfy oracle inequalities with a residual term that decreases like $1/n$ for every L_q-loss functions ($q \geq 2$). These results require only assumptions on the tail behaviour of the input and output variables. In particular, an RIP (cf. [4]) type of assumption or an incoherency condition (cf. [3]) are not needed to obtain fast residual terms in these setups. (This is a joint work with Shahar Mendelson, [6]).

We also apply these general methods to obtain exact oracle inequalities for a PERM procedure based on a penalty function using a mixture of several norms

$$\mathrm{pen}(A) = \lambda_1 \|A\|_{S_1} + \lambda_2 \|A\|_{S_2}^2 + \lambda_3 \|A\|_1$$

in the matrix completion problem. (This is a joint work with Stéphane Gaïffas, [5]).

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