U N I V E R S I T Ä T   H A M B U R G

A Polynomial

Bühlmann-Straub Method

Erhard Kremer

Preprint-No. 2009-04   Juni 2006

DEPARTEMENT MATHEMATIK

SCHWERPUNKT MATHEMATISCHE STATISTIK
UND STOCHASTISCHE PROZESSE

BUNDESSTR. 55, D – 20146 HAMBURG

A Polynomial Bühlmann-Straub-Method

Erhard Kremer (Hamburg)

1 Introduction

Credibility theorie is a very old branch of Risk Theory and/or Nonlife Insurance Mathematics. First results go back to Mowbray an Witney (1914/18), a first very elegant approach was given by Bühlmann (1967) and Bühlmann and Straub (1970). Nowadays there exist a lot of different models, in which more or less complicated credibility rating techniques are derived. The former classical models are special cases of the so-called Regression Credibility Models. Reference can be made to Bühlmann & Gisler (1997), Hachemeister (1975), Kremer (1988), (1996), Sundt (1981), (1983), Norberg (1980), (1986), Taylor (1979), and Zehnwirth (1987).

In all these models one has unknown structural parameters which have to be estimated from known claims experience. The derivation of optimal parameter estimators was subject of several publications of credibility theorists (see De Vylder (1978), Kremer (1995), Norberg (1982)).

In most papers on credibility the so-called linear credibility estimator is considered as the credibility estimator. Mathematical seen, this linear credibility estimator is an optimal linear approximation to the so-called exact credibility estimator. Theoretically seen that exact credibility estimator gives the very best experience rating procedure in the credibility model.

Papers on the exact credibility estimator are e.g. Jewell (1974), Landsman

& Markov (1998). In general the exact credibility estimator ist not linear. So it is nearlying to consider also nonlinear approximations (instead of the linear ones) to that exact credibility estimator. Most nearlying is to investigate certain polynomical approximations. That topic was the content of the paper Kremer (2006). Note the methods, given there, are not very adequate for practical rating since they do not incorporate volume measures like in the famous Bühlmann/Straub (1970) context that is important for fire rating. In the present paper an adaption of Kremer's (2006) results to the context with volume measures is given. It is possible to use the methods in practical fire rating.

2 Hilbertspace L_2

For fixed probability space $(\Omega, \mathcal{A}, P)$ let L_2 be the Hilbertspace of measurable, square-integrable functions f (identified with the equivalence class of all g which are P-a.e. equal to f) defined on Ω with the scalar product

$$\langle f, g \rangle = E(f \cdot g)$$

and the norm

$$\|f\| = [E(f^2)]^{1/2} \quad .$$

Basic is the

Defintion 1

For a closed linear subspace L of the L_2 one defines the **projection** P_L as the mapping from L_2 into L such that for each $f \in L_2$ one has

$$\|P_L(f) - f\| \leq \|f - g\|$$

for all $g \in L$. □

Obviously $P_L(f)$ is the "nearest" element of the subspaces L to the $f \in L_2$. For credibility theory there are most important the linear subspaces $L =$

$H_n(f_1, ..., f_n)$ spanned up by the $f_i \in L_2$ defined as

$$H_n(f_1, ..., f_n) = \left\{ \sum_{i=1}^{n} a_i \cdot f_i \ : \ (a_1, ..., a_n) \in \mathbb{R}^n \right\} .$$

It is well-known that fo $L = H_n(f_1, ..., f_n)$ the projection

$$P_L(f) = \sum_{i=1}^{n} \hat{a}_i \cdot f_i$$

is uniquely characterized by the so-called **normal equations**:

$$E([f - P_L(f)] \cdot f_i) = 0 , \quad \forall i = 1, ..., n .$$

3 Linear Credibility

In credibility theory one has random variables $X_i \in L_2$, $i = 1, ..., n+1$, describing the **total claims amounts** of a risk in past periods $i = 1, ..., n$ and next period $i = n+1$. Furthermore one has the random variables

$$\theta_i : (\Omega, \mathcal{A}, P) \to (\Theta, C)$$

(with $i = 1, ..., n+1$), the so-called **risk parameters** of the risk in periods $i = 1, ..., n+1$. Obviously it is assumed the so-called **collective model** of risk theory. This means that the risk is taken randomly out of a whole portfolio of (insurance) risks, modelled by assuming a random risk parameter θ_i in period no. i. For a taken risk the realization ϑ_i of θ_i characterizes the risk behaviour of the risk in period i. In most cases the ϑ_i is the parameter of the distribution of X_i for the taken risk.
One of the main problems in credibility theory is the prediction of the **net premium**

$$\begin{aligned} \mu_{n+1} &= \mu_{n+1}(\theta_i) \\ &= E(X_{n+1} | \theta_{n+1}) \end{aligned}$$

of the (randomly) taken risk (for future period no. $n+1$) based on the past claims $X_1, ..., X_n$ of the risk.

Mathematical seen the so-called **linear credibility estimator** of $\hat{\mu}_{n+1}$, usually simply called **credibility estimator**, is just the projection

$$\hat{\mu}_{n+1} = P_L(\mu_{n+1})$$

with

$$L = H_n(1, X_1, ..., X_n) \tag{3.1}$$

where 1 is the constant function, taking on only the value 1. The normal equations for $\hat{\mu}_{n+1}$ are equivalent with

$$\begin{aligned} E(\hat{\mu}_{n+1}) &= E(\mu_{n+1}) \\ Cov(\hat{\mu}_{n+1}, X_i) &= Cov(\mu_{n+1}, X_i)\,, \quad i = 1, ..., n\,. \end{aligned} \tag{3.2}$$

Inserting the formula

$$\hat{\mu}_{n+1} = \hat{a}_0 + \sum_{i=1}^{n} \hat{a}_i \cdot X_i$$

gives

$$\hat{a}_0 = E(\mu_{n+1}) - \sum_{i=1}^{n} \hat{a}_i \cdot E(X_i) \tag{3.3}$$

with the solutions $\hat{a}_i$, $i = 1, ..., n$ of linear equation system

$$\sum_{j=1}^{n} Cov(X_j, X_i)\hat{a}_j = Cov(\mu_{n+1}, X_i) \quad i = 1, ..., n\,. \tag{3.4}$$

The so determined credibility estimator $\hat{\mu}_{n+1}$ is the optimal linear-affine prediction of μ_{n+1} based on $X_1, ..., X_n$.

4 Polynomial Credibility

Those linear-affine approximations of section 3 are clearly quite handy, but on the other side in general not the best ones. A quite nearlying improvement is to replace the L of (3.1) by the

$$L = H_{nr+1}(1, g_1(X_1), ..., g_r(X_1), ..., g_1(X_n), ..., g_r(X_n)) \tag{4.1}$$

with the functions

$$g_p(x) = x^p \,, \quad p = 1, ..., r \,, \tag{4.2}$$

and assuming that

$$g_p(X_i) \in L_2 \,, \quad \begin{aligned} i &= 1, ..., n \\ p &= 1, ..., r \,. \end{aligned} \tag{4.3}$$

The projection

$$\hat{\hat{\mu}}_{n+1} = P_L(\mu_{n+1})$$

can be called **polynomial credibility estimator** of μ_{n+1}. The equations (3.2) change to

$$\begin{aligned} E(\hat{\hat{\mu}}_{n+1}) &= E(\mu_{n+1}) \\ Cov(\hat{\hat{\mu}}_{n+1}, g_p(X_i)) &= Cov(\mu_{n+1}, g_p(X_i)) \end{aligned} \tag{4.4}$$

$i = 1, ..., n$, $p = 1, ..., r$;
and the coefficients of

$$\hat{\hat{\mu}}_{n+1} = \hat{a}_0 + \sum_{i=1}^{n} \left(\sum_{p=1}^{r} \hat{a}_{ip} \cdot g_p(X_i) \right) \tag{4.5}$$

are

$$\hat{a}_0 = E(\mu_{n+1}) - \sum_{i=1}^{n} \left(\sum_{p=1}^{r} \hat{a}_{ip} \cdot E(g_p(X_i)) \right) \tag{4.6}$$

with $\hat{a}_1, ..., \hat{a}_r$ the solutions of

$$\sum_{j=1}^{n} \sum_{p=1}^{r} \hat{a}_{jp} \cdot Cov(g_p(X_j), g_p(X_i)) = Cov(\mu_{n+1}, g_p(X_i)) \qquad \text{for } p = 1, ..., r \,,\ i = 1, ..., n \,. \tag{4.7}$$

Remember thet these results were already given in Kremer (2006).
Assume more special for the sequel:

$$\boxed{\theta_i = \theta}, \quad i = 1, ..., n \tag{4.8}$$

5 Empirical Polynomial Credibility

For applying (4.5) with (4.6), (4.7) one needs estimators for the means

$$\begin{aligned} \nu_{n+1} &= E(\mu_{n+1}) \\ \nu_p(i) &= E(g_p(X_i)) \,, \quad i = 1, ..., n \\ & \qquad\qquad\qquad\quad p = 1, ..., r \end{aligned}$$

and the covariances

$$\begin{aligned} b_p(i) &= Cov(\mu_{n+1}, g_p(X_i)) \\ c_{pq}(i,j) &= Cov(g_p(X_i), g_q(X_j)) \end{aligned}$$

for $i = 1, ..., n$, $q, p = 1, ..., r$. For the sequel assume in addition (4.8) and

$$\boxed{\mu_i(\theta) = \mu(\theta)} \,, \quad \forall i = 1, ..., n+1 \tag{5.1}$$

for

$$\mu_i(\theta) = E(X_i|\theta)$$

with an unknown measurable functions

$$\mu : \Theta \to [0, \infty) \,.$$

The assumption (5.1) implies

$$\nu_1(i) = \nu_{n+1} = \nu \quad \forall i = 1, ..., n$$

with the **collective net premium**

$$\nu = E(\mu(\theta)) \,.$$

Assume now, that one has the claims data of the whole collective of risks. Let K be the number of risks in the collective. Denote with X_{ki} the total claims amount of risk no. k in past period no. i and with θ_i ($= \theta$ for risk no. k under (4.8)!) the (random) risk parameter of that risk no. k. Suppose that (5.1) (with $\theta = \theta_k$) holds for each risk of the collective and that the vectors

$$(\theta_k, X_{k1}, ..., X_{kn}), \quad k = 1, ..., K$$

are independent. In Kremer (2006) quite simple model assumptions were choosen under which reasonable estimators $\hat{\nu}$, $\hat{\nu}_p(i)$, $\hat{b}_p(i)$, $\hat{c}_{pq}(i,j)$ of ν, $\nu_p(i)$, $b_p(i)$, $c_{pq}(i,j)$ could be given.
Inserting the estimators $\hat{c}_{pq}(i,j)$, $\hat{b}_p(i)$ for the $c_{pq}(i,j)$, $b_p(i)$ into (4.7) one gets (after solving the system) as solutions the empirical coefficients $\hat{a}^e_{j,q}$, $q = 1, ..., r$, $j = 1, ..., n$. Putting them and $\hat{\nu}$ (for $E(\mu_{n+1})$), $\hat{\nu}_p(i)$ (for $\nu_p(i)$) into (4.6) one gets the empirical $\hat{a}^e_0$. Altogether

$$\hat{\mu}^e_{n+1} = \hat{a}^e_0 + \sum_{i=1}^{n} \left(\sum_{p=1}^{r} \hat{a}^e_{ip} \cdot X^p_i \right) \tag{5.2}$$

(with $(X_1, ..., X_n) = (X_{k1}, ..., X_{kn})$ for risk no. k) gives what the author called the **empirical polynomial credibility estimator** for $\mu(\theta) = \mu_{n+1}$. With this one has the complete polynomial credibility rating procedure.

6 More Refined Case

The simple model assumptions in Kremer (2006) are usually not given. So for example not in the typical case of fire rating like e.g. in Wenger (1973) or Kremer (1985). One works there with socalled volume measures. The classical credibility-method in that situation is that in Bühlmann & Straub (1970). Bühlmann & Straub's credibility estimator is a linear one. Certainly one likes to have also something like a polynomial version. To get something adequate handy, one needs further ideal model assumptions. So suppose that the conditional distributions of X_i given $\theta = \vartheta$ has the Lebesgue density:

$$f_i(x|\vartheta) = g_i(x - \vartheta)$$

with

$$g_i(x) = \frac{1}{\sqrt{2\pi}} \cdot \frac{\sqrt{V_i}}{\sigma} \cdot \exp\left(-\frac{V_i}{2\sigma^2} \cdot x^2 \right) \tag{6.1}$$

where V_i is the given volume measure in period no. i. Note that according to Flach et al. (1970) one can reach in fire rating by truncation that the X_i are given $\theta = \vartheta$ approximately normal-distributed (like according to (6.1)). For

the moment suppose that also σ^2 is given.
Note that under (6.1)

$$\mu(\theta) = E(X_i|\theta) = \theta$$
$$Var(X_i|\theta) = \frac{\sigma^2}{V_i}$$

These basic ideas have to be carried over to the context of a collective of risks like in part 5 of the paper. So suppose again that the vectors

$$(\theta_k, X_{k1}..., X_{kn}), \quad k = 1, ..., K$$

of the K risks of the collective are independent. Furthermore assume that the θ_k are all distributed like θ, but that the conditional distribution of X_{ki} given $\theta_k = \vartheta$ has a Lebesgue-density:

$$f_{ki}(x|\vartheta) = g_{ki}(x - \vartheta)$$

with in analogy to (6.1):

$$g_{ki}(x) = \frac{1}{\sqrt{2\pi}} \cdot \frac{\sqrt{V_{ki}}}{\sigma} \cdot \exp\left(-\frac{V_{ki}}{2\sigma^2} \cdot x^2\right) , \tag{6.2}$$

where the V_{ki} is the given volume measure of risk no. k in period i. For the moment let σ^2 be given. Note that again

$$E(X_{ki}|\theta_k = \vartheta) = \vartheta$$
$$Var(X_{ki}|\theta_k = \vartheta) = \frac{\sigma^2}{V_{ki}}$$

Finally suppose that:

$$Cov(X_{ki}^p, X_{kj}^q|\theta_k = \vartheta) = \frac{\nu_{pq}(i,j)}{V_{ki}^{p/2} V_{kj}^{q/2}} \tag{6.3}$$

with unknown $\nu_p(i, j)$ and all natural k, p, q.
Note that this condition follows from (6.2) always for case $i = j$ with

$\nu_{pq}(i,i) = \nu_{pq}$ independent of i. Under these model assumptions the estimators of section 5 in Kremer (2006) have to be modified considerably. It is obvious to estimate $\nu = \nu_1(i) = E(X_i)$ by

$$\hat{\nu} = \sum_{k=1}^{K} \sum_{i=1}^{n} \left(\frac{V_{ki}}{V_{..}} \right) \cdot X_{ki} \tag{6.4}$$

with

$$V_{..} = \sum_{k=1}^{K} \sum_{i=1}^{n} V_{ki} \quad .$$

Furthermore it is obvious to estimate $\nu_{pq}(i,j)$ of (6.3) according

$$\text{for } i \neq j: \quad \hat{\nu}_{pq}(i,j) = \frac{1}{K} \cdot \sum_{k=1}^{K} V_{ki}^{p/2} \cdot V_{kj}^{q/2} \cdot (X_{ki}^p - \hat{\nu}_p^{(k)}(i)) \cdot (X_{kj}^p - \hat{\nu}_q^{(k)}(j)) \ ,$$

$$\text{for } i = j: \quad \hat{\nu}_{pq}(i,i) = \frac{1}{K} \cdot \sum_{k=1}^{K} \sum_{i=1}^{n} V_{ki}^{(p+q)/2} \cdot (X_{ki}^p - \hat{\nu}_p^{(k)}(i)) \cdot (X_{ki}^p - \hat{\nu}_q^{(k)}(i))$$

where $\hat{\nu}_p^{(k)}(i)$ is an estimator for:

$$\nu_p^{(k)}(i) = E(X_{ki}^p | \theta_k)$$

From (6.2) it follows (with (4.2) in Pensky & Ni (2000)) that there exists a $\nu_p > 0$, independent of k, i, with

$$E((X_{ki} - \nu_1^{(k)}(i))^p | \theta_k = \vartheta) = \frac{\nu_p}{V_{ki}^{p/2}}$$

Obviously a senseful estimator of ν_p is

$$\hat{\nu}_p = \frac{1}{K \cdot n} \cdot \sum_{k=1}^{K} \sum_{i=1}^{n} V_{ki}^{p/2} \cdot (X_{ki} - \hat{\nu}_1^{(k)})^p$$

where $\nu_1^{(k)}(i)$ is estimated by

$$\hat{\nu}_1^{(k)} = \sum_{j=1}^{n} \frac{V_{kj}}{V_{k\cdot}} \cdot X_{kj}$$

with

$$V_{k\cdot} = \sum_{i=1}^{n} V_{ki} \quad .$$

Since

$$E(X_{ki} - \nu_1^{(k)}(i)) = \sum_{j=0}^{p} \binom{p}{j} \cdot E(X_{ki}^j|\theta_k = \vartheta) \cdot (-\nu_1^{(k)}(i))^{p-j}$$

implying

$$E(X_{ki}^p|\theta_k = \vartheta) = \nu_p/(V_{ki}^{p/2}) - \sum_{j=0}^{p-1} \binom{p}{j} \cdot E(X_{ki}^j|\theta_k = \vartheta) \cdot (-\nu_1^{(k)}(i))^{p-j} \ ,$$

one gets as recursion for an estimator $\hat{\nu}_p^{(k)}(i)$ for $\nu_p^{(k)}(i)$ the following

$$\hat{\nu}_p^{(k)}(i) = \frac{\hat{\nu}_p}{V_{ki}^{p/2}} - \sum_{j=0}^{p-1} \binom{p}{j} \cdot \hat{\nu}_j^{(k)}(i) \cdot (-\nu_1^{(k)})^{p-j} \qquad (6.5)$$

with start

$$\hat{\nu}_0^k(i) = 1 \ .$$

Combining formula (5.6) in Kremer (2006) with now

$$\Psi_{pi}(x) = x^{p+1} - p \cdot \frac{\sigma^2}{V_{ki}} \cdot x^{p-1} \qquad p = 1, ..., r \ , \ i = 1, ..., n$$

(compare Theorem 2 in Pensky & Ni (2002)), one arrives as the estimator for

$$b_p^{(k)}(i) = Cov(\mu_{n+1}, X_{ki}^p)$$

at

$$\hat{b}_p^{(k)}(i) = \hat{\nu}_{p+1}^{(k)}(i) - p \cdot \frac{\sigma^2}{V_{ki}} \cdot \hat{\nu}_{p-1}^{(k)}(i) - \hat{\nu}_p^{(k)}(i) \cdot \hat{\nu} \ . \qquad (6.6)$$

Furthermore (because of (6.3)) it results as estimator of

$$Cov(X_{kj}^q, X_{ki}^p)$$

the

$$\hat{c}_{pq}(i,j) = \frac{\hat{\nu}_{pq}(i,j)}{V_{ki}^{p/2} \cdot V_{kj}^{q/2}} \tag{6.7a}$$

in case $i \neq j$. In the case $i = j$ one has to consider the decomposition (in analogy of Kremer (1999), Lemma 3.11(a)):

$$Cov(X_{ki}^p, X_{ki}^q) = \alpha_{pq}(i) + \beta_{pq}(i)$$

with

$$\begin{aligned} \alpha_{pq}(i) &= E(Cov(X_{ki}^p, X_{ki}^q | \theta_k)) \\ \beta_{pq}(i) &= Cov(E(X_{ki}^p | \theta_k), E(X_{ki}^q | \theta_k)) \ . \end{aligned}$$

In analogy to (6.7a) one can give as estimator of $\alpha_{pq}(i)$

$$\hat{\alpha}_{pq}(i) = \frac{\hat{\nu}_{pq}(i,i)}{V_{ki}^{(p+q)/2}} \quad ,$$

and nearlying as estimator of $\beta_{pq}(i)$

$$\hat{\beta}_{pq}(i) = \frac{\hat{\eta}_{pq}}{V_{ki}^{(p+q)/2}} \quad ,$$

with

$$\hat{\eta}_{pq} = \frac{1}{K \cdot n} \cdot \sum_{k=1}^{K} \sum_{i=1}^{n} V_{ki}^{(p+q)/2} \cdot (\hat{\nu}_p^{(k)}(i) - \hat{\nu}_p^{(k)}) \cdot (\hat{\nu}_q^{(k)}(i) - \hat{\nu}_q^{(k)})$$

where

$$\begin{aligned} \hat{\nu}_p^{(k)} &= \sum_{i=1}^{n} \frac{V_{ki}^p}{V_{k\cdot}^{(p)}} \cdot X_{ki}^p \\ V_{k\cdot}^{(p)} &= \sum_{i=1}^{n} V_{ki}^p \end{aligned}$$

(analogously for p replaced by q).
Putting all things together one arrives at the sensefull estimator of $Cov(X_{ki}^p, X_{ki}^q)$

$$\hat{c}_{pq}(i,i) = \hat{\alpha}_{pq}(i) + \hat{\beta}_{pq}(i) \tag{6.7b}$$

Only one thing is open. What is with σ^2? The author proposes to estimate σ^2 by

$$\hat{\sigma}^2 = \frac{n}{n-1} \cdot \hat{\nu}_2$$

This $\hat{\sigma}^2$ has to be inserted into the rhs of (6.6) and with this new (6.6) one arrives at the final estimators

$$\hat{\hat{b}}_p^{(k)}(i) \; .$$

The whole **new practical rating procedure** works as follows:

1. Insert $\hat{\hat{b}}_p^{(k)}(i)$ for $Cov(\mu_{n+1}, g_p(X_i))$ and $\hat{c}_{pq}^{(k)}$ (according to (6.7 a+b)) for $Cov(g_q(X_j), g_p(X_i))$ into (4.7). Solve the system, giving the solutions $\hat{a}_{jp}^e = \hat{a}_{jp}^e(k)$, $p = 1, ..., r$, $j = 1, ..., n$.

2. Insert $\hat{\nu}$ (according to (6.4)) for $E(\mu_{n+1})$ and $\hat{\nu}^{(k)}(i)$ (according to (6.5)) for $E(g_p(X_i))$ into (4.6). With inserted $\hat{a}_{ip} = \hat{a}_{ip}^e(k)$, $i = 1, ..., n$, $p = 1, ..., r$ one arrives at the estimate $\hat{a}_0^e(k)$.

3. Then rate μ_{n+1} for risk no. k with the empirical formula

$$\hat{\mu}_{k+1}^e(k) = \hat{a}_0^e(k) + \sum_{i=1}^{n} \left(\sum_{p=1}^{r} \hat{a}_{ip}^e(k) \cdot X_{ki}^p \right) \; .$$

7 Final Remarks

The author guesses that for fire-practice it is most adequate to choose $p = 3$. Furthermore note that in the model of section 6 ist is <u>not</u> assumed that

$$X_{k1}, ..., X_{kn} \text{ are given } \theta_k \text{ independent,} \tag{7.1}$$

what is usually taken in papers on credibility methods. The above assumption (6.3) is quite more general. In case of more simple (7.1) one only has to put in (6.3):

$$\nu_{pq}(i,j) = 0 \quad \forall i \neq j$$

and consequently:

$$\hat{\nu}_{pq}(i,j) = 0 \quad \forall i \neq j \ ,$$

giving in (6.7a)

$$\hat{c}_{pq}(i,j) = 0 \quad \forall i \neq j \ .$$

□

With more general (6.3) (without (7.1)) also situations of evolutionary credibility models are incorporated.

References

[1] **Bühlmann, H. (1967):** Experience rating and credibility. *ASTIN Bulletin.*

[2] **Bühlmann, H. & Straub, E. (1970):** Glaubwürdigkeit für Schadensätze. *Mitteilungen der Vereinigung Schweizerischer Versicherungsmathematiker.*

[3] **Bühlmann, H. & Gisler, A. (1977):** Credibility in the regression case revisited. *ASTIN Bulletin*

[4] **De Vylder, F. (1978):** Parameter estimation in credibility thery. *ASTIN Bulletin*

[5] **De Vylder, F. & Goovaerts, M. (1985):** Semilinear credibility with several approximating functions. *Insurance: Mathematics & Economics.*

[6] **Flach, D. und Strauß, J. (1970):** Analyse der deutschen Feuer-Industrie-Statistik. *Blätter der Deutschen Gesellschaft für Versicherungsmathematik.*

[7] **Hachemeister, C.A. (1975):** Credibility for regression models with application to trend. *Credibility: Theory & Applications. Academic Press, New York.*

[8] **Jewell, W.S. (1974):** Regularity conditions for exact credibility. *ASTIN Bulletin*

[9] **Kremer, E. (1985):** Einführung in die Versicherungsmathematik. *Vandenhoeck & Ruprecht, Göttingen.*

[10] **Kremer, E. (1988):** Box-Jenkins Credibility. *Blätter der Deutschen Gesellschaft für Versicherungsmathematik.*

[11] **Kremer, E. (1995):** Empirical Kalman credibility.*Blätter der Deutschen Gesellschaft für Versicherungsmathematik.*

[12] **Kremer, E. (1996):** Credibility for stationarity. *Blätter der Deutschen Gesellschaft für Versicherungsmathematik.*

[13] **Kremer, E. (1999):** Applied Risk Theory. *Shaker, Aachen.*

[14] **Kremer, E. (2006):** Polynomial Credibility. *Blätter der Deutschen Gesellschaft für Versicherungsmathematik.*

[15] **Landsman, Z.M. and Makov, U.E. (1998):** Exponential dispersion models and credibility. *Scandinavian Actuarial Journal.*

[16] **Mowbray, A.H. (1914):** How extensive a payroll exposure is necessary to give a dependable pure premium. *Proceedings of the Casualty Actuarial Society.*

[17] **Norberg, R. (1980):** Empirical Bayes credibility. *Scandinavian Actuarial Journal.*

[18] **Norberg, R. (1982):** On optimal parameter estimation in credibility. *Insurance: Mathematics & Economics.*

[19] **Norberg, R. (1986):** Hierarchical credibility: Analysis of a random effect linear model with nested classification. *Scandinavian Actuarial Journal.*

[20] **Pensky, M. & Ni, P. (2000):** Extended linear empirical Bayes estimation. *Communications in Statistics: Theory and Methods.*

[21] **Sundt, B. (1981):** Recursive credibility estimation. *Scandinavian Actuarial Journal.*

[22] **Sundt, B. (1983):** Finite credibility formulae in evolutionary models. *Scandinavian Actuarial Journal.*

[23] **Taylor, G.C. (1979):** Credibility analysis of a general hierachical model. *Scandinavian Actuarial Journal.*

[24] **Wenger, H. (1973):** Eine Tarifierungsmethode im Feuer-Industrie-Geschäft. *Mitteilungen der Vereinigung Schweizerischer Versicherungsmathematiker.*

[25] **Witney, A.W. (1918):** The theory of experience rating. *Proceedings of the Casualty Actuarial Society.*

[26] **Zehnwirth, B. (1985):** Linear filtering and recursive credibility estimation. *ASTIN, Bulletin.*

Summary

The concepts of polynomial credibility, as published in 2006 by the author, is adapted to a situation of the Bühlmann & Straub - model, like it can occur in fire premium rating.