Matroid theory: exercise sheet 1

1. For which numbers m, n is the uniform matroid $U_{m,n}$ graphic?

2. For which numbers m, n is the uniform matroid $U_{m,n}$ representable over the field $\mathbb{F}_2$ with 2 elements? For which numbers is it representable over the field $\mathbb{R}$ of real numbers.

3. Let M be a matroid on E. For $X \subseteq E$, the *closure* $\mathrm{Cl}_M(X)$ of X is the set

$$\{x \in E | r_M(X \cup x) = r_M(X)\}$$

   Prove that the function $\mathrm{Cl} := \mathrm{Cl}_M : \mathcal{P}E \to \mathcal{P}E$ has the following properties:

   (CL1) For any $X \subseteq E$ we have $X \subseteq \mathrm{Cl}(X)$

   (CL2) For any $X \subseteq Y \subseteq E$ we have $\mathrm{Cl}(X) \subseteq \mathrm{Cl}(Y)$

   (CL3) For any $X \subseteq E$ we have $\mathrm{Cl}(\mathrm{Cl}(X)) = \mathrm{Cl}(X)$

   (CL4) For any $X \subseteq E$, $x \in E$ und $y \in \mathrm{Cl}(X \cup x) - \mathrm{Cl}(X)$ we have $x \in \mathrm{Cl}(X \cup y)$

4.* Prove that a function $\mathrm{Cl}\colon \mathcal{P}E \to \mathcal{P}E$ is the closure operator of a matroid on E if and only if it satisfies(CL1)-(CL4).