Matroidentheorie: Übungsblatt 3

1. Let M be a matroid which is representable over the field $\mathbb{F}_2$ with two elements. Let C be a circuit and D a cocircuit of M. Prove that $|C \cap D|$ is even.

2. A *signing* of a matroid M consists of functions $f_C \colon C \to \{\pm 1\}$ for $C \in \mathcal{C}(M)$ and $g_D \colon D \to \{\pm 1\}$ for $D \in \mathcal{C}^*(M)$ such that for any $C \in \mathcal{C}(M)$ and $D \in \mathcal{C}^*(M)$ we have $\sum_{e \in C \cap D} f_C(e) g_D(e) = 0$. Let M be a matroid which has a signing. Prove that M is regular (representable over every field).

3. Let E be a finite set and V a subspace of k^E. Let P and Q be disjoint sets, and let $V/P\backslash Q$ be the subspace $\{v\restriction_{E-P-Q} \colon v \in V \text{ and } \underline{v} \cap Q = \emptyset\}$ of k^{E-P-Q}. Prove that $M(V)/P\backslash Q = M(V/P\backslash Q)$.

4.* (a) Let P be a subset of the ground set E of a matroid M and let B be a basis of P in M. Prove that $M/P = M/B\backslash(P - B)$.

(b) Let N be a minor of a matroid M. Prove that there are an independent set P and a coindependent set Q of M with $N = M/P\backslash Q$.