## Matroid theory: exercise sheet 4

- 1. Find as many matroids on a two-element set as you can. For each such matroid N, find a simple characterisation of the matroids with no N-minor. [1 point for each such matroid N, up to duality]
- 2. Let M be a connected matroid on E with  $|E| \ge 2$  and let  $e \in E$ . Prove that at least one of  $M/\{e\}$  and  $M\setminus\{e\}$  is connected.
- 3.\* Let M be a matroid on E and B a basis of M. Let G be the bipartite graph on B and E-B with an edge from  $e \in B$  to  $f \in E-B$  when  $e \in C_f^B$ . Prove that M is connected as a matroid if and only if G is connected as a graph.