Matroid theory: Exercise sheet 5

1. A matroid is *infinitely connected* if it is n-connected for all $n \in \mathbb{N}$. Prove that every infinitely connected matroid is uniform.

2. Let M be a matroid on E, let $X \subseteq E$ and let $e \in E - X$. Show that $\kappa_M(X + e) = \kappa_M(x)$ if and only if e lies in exactly one of the closure and the coclosure of X in M.

3. Let G be a connected graph and let A and B be edge-disjoint connected subgraphs of G. Prove that the minimal size of a set $S \subseteq V(G)$ meeting all paths from A to B is precisely
$$\min_{E(A) \subseteq X \subseteq E(G) - E(B)} \kappa_{M(G)}(X) + 1\,.$$

4. Let G be a connected graph and let A and B be edge-disjoint connected subgraphs of G. Let $F = E - (E(A) \cup E(B))$. Show that the maximum number of disjoint paths from A to B is the same as the maximum over all partitions $F = P \dot\cup Q$ of $\kappa_{M/P \backslash Q}(E(A)) + 1$.