# Matroid theory: exercise sheet 8

1. Let $M$ and $M'$ be binary matroids on the same set $E$ such that they have a common basis $B$ and, for any element not in $B$, the fundamental circuits of that element with respect to $B$ in $M$ and $M'$ are the same. Show that $M = M'$.

2. Let $M$ be a binary matroid with a circuit $C$ and a cocircuit $D$ such that $|C \cap D| = 4$. Prove that $M$ has an $M(K^4)$-minor.

3. Prove that a matroid is binary if and only if it has the following property: for any two bases $B_1$ and $B_2$ and any $x \in B_2 - B_1$ the set

$$\{y \in B_1 - B_2 | B_1 - y \cup x \text{ und } B_2 - x \cup y \text{ are bases}\}$$

has odd size.

4. Prove that a matroid is binary if and only if the set $\mathcal{C}$ of its circuits satisfies the following stronger elimination axiom:

$(C3)_{\text{bin}}$ For any $C \in \mathcal{C}$, distinct elements $z$, $x$ and $y$ of $\mathcal{C}$, and $C_x, C_y \in \mathcal{C}$ such that $x \in C_x$ and $y \in C_y$ but $z$ is neither in $C_x$ nor $C_y$, there is $C' \in \mathcal{C}$ with $z \in C' \subseteq (C \cup C_x \cup C_y) \setminus \{x, y\}$.