Matroid theory: exercise sheet 9

1. Find all 3-connected binary matroids with 8 elements.

2. Using the previous exercise or otherwise, show that F_7 is a splitter for the class of binary matroids with no F_7^*-minor.

3. Let M_1 and M_2 be binary matroids on sets E_1 and E_2 with $|E_1 \cap E_2| = k$. Show that $\kappa_{M_1 \oplus_{\mathbb{F}_2} M_2}(E_1 - E_2) = k$ if and only if $E_1 \cap E_2$ is both independent and coindependent in both M_1 and M_2.

4* Let M be a binary matroid with ground set E and let $X \subseteq E$ with $\kappa_M(X) = k$. Show that there are a set G disjoint from E of size k and binary matroids M_1 on $X \cup G$ and M_2 on $(E - X) \cup G$ with $M_1 \oplus_{\mathbb{F}_2} M_2 = M$.