Seminar: Higher Algebra

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Meeting time: Tuesdays 12:15-13:45

Location: SR 0.007

Description: Let X be a connected topological space and let k be a field. Denote the differential graded algebra of k-linear chains on the based loop space of X by $C_\bullet(\Omega X; k)$. Goodwillie[1] and Jones[2] provided the formula

$$\mathrm{HH}_*(C_\bullet(\Omega X; k) \cong \mathrm{H}_*(LX; k)$$

relating the Hochschild homology of this dg algebra to the homology of the free loop space LX. The goal of this seminar will be to understand a generalization of this result in the higher categorical context of E_n-algebras called "nonabelian Poincaré duality". Assuming familiarity with [3], the seminar will cover the material from [4], necessary to understand the statement and the rudiments of its proof.

Talk 1. (Apr 19, Toby) Introduction. Hochschild homology for associative and commutative algebras, classical result of Goodwillie and Jones, cyclic category, higher Hochschild homology.

Talk 2. (Apr 25, Giulio) Presentable ∞-categories. Focus on sections 5.5.2, 5.5.3 in [3], Prove the adjoint functor theorem.

Talk 3. (May 2, Fernando) Symmetric monoidal ∞-categories. Explain the formulation of symmetric monoidal categories in terms of fibrations over $\mathcal{F}\mathrm{in}_*$ (Introduction to Cahpter 2), introduce symmetric monoidal ∞-categories (2.0.0.7), Examples: Cartesian monoidal structures (2.4.1.4), nerves of symmetric monoidal model categories (4.1.3.10).

Talk 4. (May 9, Jack) Classical operads. Define planar and symmetric operads in a symmetric monoidal category, Examples (associative, commutative, Lie, endomorphism), define algebras over operad (e.g. [5]) and discuss further topics according to taste.

Talk 5. (May 16, Tashi) ∞-operads. Define ∞-operads providing short motivation via colored operads (2.1.1), discuss the various notions of fibrations (2.1.2), algebra objects (2.1.3), and the model category of preoperads (2.1.4).

Talk 6. (May 23, Tina) Constructions of ∞-operads. Discuss the classical Boardman-Vogt tensor product [7], and discuss its generalization to ∞-operads explaining the results in 2.2.5.

Talk 7. (June 13, Stefano) Algebras and modules. Explain the notion of a module over an algebra over a coherent ∞-operad (3.3.3.8 and 3.3.3.9), state 3.4.4.2 and survey a selection of ideas which enter its proof.

Talk 8. (June 20, Michael) Associative algebras. Introduce the operad $\mathcal{A}\mathrm{ss}^{\otimes}$, planar ∞-operads (4.1.1), the notion of a left module (4.2.1), and discuss the simplicial model in 4.7.1.

Talk 9. (June 27, Axel) E_n-operads. Introduce the classical little cubes operas and discuss May's delooping theorem [6]. Introduce E_n-∞-operads, and show how they interpolate between $\mathcal{A}\mathrm{ss}^{\otimes}$ and $\mathcal{C}\mathrm{omm}^{\otimes}$ (5.1.0.7 and 5.1.1.5), sketch 5.1.1.1.

Talk 10. (July 4, Felix) Additivity theorem. Sketch the proof of 5.1.2.2 and explain 5.1.2.3 and 5.1.2.4.

Talk 11. (July 11, Gustavo) Little cubes in manifolds. Survey 5.4.

Talk 12. (July 18, Walker) Topological chiral homology. Focus on 5.5.1 and 5.5.2, and mention properties in 5.5.3.

Talk 13. (July 25, Toby) Nonabelian Poincaré duality. Discuss 5.5.6.6.

References

[1] Thomas G Goodwillie. Cyclic homology, derivations, and the free loopspace. *Topology*, 24(2):187–215, 1985.

[2] John DS Jones. Cyclic homology and equivariant homology. *Inventiones mathematicae*, 87(2):403–423, 1987.

[3] Jacob Lurie. *Higher topos theory*. Number 170. Princeton University Press, 2009.

[4] Jacob Lurie. Higher algebra. 2014. *Preprint, available at http://www. math. harvard. edu/˜ lurie*, 2016.

[5] Martin Markl, Steven Shnider, and James D Stasheff. *Operads in algebra, topology and physics*. Number 96. American Mathematical Soc., 2007.

[6] J Peter May. *The geometry of iterated loop spaces*, volume 271. Springer, 2006.

[7] Ittay Weiss. From operads to dendroidal sets. *Mathematical foundations of quantum field theory and perturbative string theory*, 83:31–70, 2011.