Seminar: Factorization homology

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Meeting time: Monday 10:15 - 11:45 (Geom 432)

Organizational Meeting: April 1, 2019

Description: We will explore E_n-algebras and factorization homology with a view towards applications to topological field theories. The main references will be [6] for the introductory part and [2] for the later part.

Talk 1. (Apr 01, Tobias) *Introduction and organization.*

Talk 2. (Apr 08) *Classical operads.* Define planar and symmetric operads in a symmetric monoidal category (e.g. [7, 1.2] or [4]). Examples: associative, commutative, Lie, endomorphism. Define algebras over operad (e.g. [7] or [4]) and relate to familar cases. Define colored operads (e.g. [6, 2.1.1]). Examples: colored operad of a symmetric monoidal category, two-colored operad governing algebra+module, etc.

Talk 3. (Apr 15) *Symmetric monoidal ∞-categories.* Explain the description of symmetric monoidal categories in terms of fibrations over $\mathcal{F}\mathrm{in}_*$ (Introduction to Chapter 2), introduce symmetric monoidal ∞-categories (2.0.0.7), relate to monoid objects in Cat and Cat_∞. Examples: Cartesian monoidal structures (2.4.1.4), nerves of symmetric monoidal model categories (4.1.3.10).

Talk 4. (Apr 29) *∞-operads.* Define ∞-operads and relate to colored operads (2.1.1), discuss the various notions of fibrations (2.1.2), algebra objects (2.1.3), and the model category of preoperads (2.1.4). Examples: $\mathcal{A}\mathrm{ss}$ (4.1.1.3), and examples 2.1.1.18-2.1.1.21.

Talk 5. (May 06) *E_n-operads.* Introduce the classical little cubes operads and discuss May's delooping theorem [8]. Introduce E_n-∞-operads, and explain how they interpolate between $\mathcal{A}\mathrm{ss}^\otimes$ and $\mathcal{C}\mathrm{omm}^\otimes$ (5.1.0.7 and 5.1.1.5), sketch 5.1.1.1.

Talk 6. (May 13) *Little cubes in Manifolds* Survey 5.4.1 and 5.4.2. Introduce the operads $\mathbb{E}_M^\otimes$ and $N(\mathrm{Disk}(M))^\otimes$ (5.4.5), and discuss their relation to the $\mathbb{E}_k$-operads. Discuss the relation between $N(\mathrm{Disk}(M))^\otimes$-algebras and $\mathbb{E}_M^\otimes$-algebras (5.4.5).

Talk 7. (May 20) *Topological chiral homology.* Explain the construction of the topological chiral homology of a manifold M (5.5.1-5.5.2). Discuss remark 5.5.2.9 and 5.5.2.10. Survey the properties from 5.5.3.

Talk 8. (May 27) Models for (∞, n)-categories [9, 5]…

Talk 9. (Jun 03) The cobordism hypothesis [5, 3]...

Talk 10. (Jun 17) Factorization homology and extended TFTs [10]...

Talk 11. (Jun 24) Factorization homology for (∞, n)-categories I [2]

Talk 12. (Jul 01) Factorization homology for (∞, n)-categories II [2]

Talk 13. (Jul 08) The cobordism hypothesis II [1]

References

[1] David Ayala and John Francis. The cobordism hypothesis. *arXiv preprint arXiv:1705.02240*, 2017.

[2] David Ayala, John Francis, and Nick Rozenblyum. Factorization homology I: Higher categories. *Adv. Math.*, 333:1042–1177, 2018.

[3] Damien Calaque and Claudia Scheimbauer. A note on the (infty, n)-category of cobordisms. *arXiv preprint arXiv:1509.08906*, 2015.

[4] Jean-Louis Loday and Bruno Vallette. *Algebraic operads*, volume 346 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer, Heidelberg, 2012.

[5] Jacob Lurie. On the classification of topological field theories. In *Current developments in mathematics, 2008*, pages 129–280. Int. Press, Somerville, MA, 2009.

[6] Jacob Lurie. Higher algebra. 2019. *Preprint, available at http://www. math. harvard. edu/˜ lurie*, 2019.

[7] Martin Markl, Steven Shnider, and James D Stasheff. *Operads in algebra, topology and physics*. Number 96. American Mathematical Soc., 2007.

[8] J Peter May. *The geometry of iterated loop spaces*, volume 271. Springer, 2006.

[9] Charles Rezk. A Cartesian presentation of weak n-categories. *Geom. Topol.*, 14(1):521–571, 2010.

[10] Claudia Scheimbauer. Factorization homology as a fully extended topological field theory. *thesis*, 2014.

[11] Graeme Segal. Configuration-spaces and iterated loop-spaces. *Invent. Math.*, 21:213–221, 1973.