

PROPOSED SCHEDULE FOR HIGHER STRUCTURES SEMINAR ON SYMPLECTIC COHOMOLOGY

(1) Lecture 1:

- Symplectic and contact manifolds definitions and examples. Exact symplectic manifolds, symplectisation of contact manifolds. Compatible (almost) complex structures.
- Reeb vector fields. Reeb orbits.
- Liouville domains.

This would be suitable for a master's student or beginning PhD student. Possible references include

- Chapters 1 and 2 of <https://people.math.ethz.ch/~acannas/Papers/lsg.pdf>
- Chapters 3 and 4 of 'Introduction to symplectic topology' by McDuff and Salamon.
- A nice reference for contact geometry is:
<https://etnyre.math.gatech.edu/preprints/papers/phys.pdf>.

(2) Lecture 2:

- Liouville manifolds.
- Examples/ non examples of all of the above. How does symplectic topology differ from topology? E.g., explain how $T^*\mathbb{R}$ and \mathbb{R}^2 are not the same as Liouville manifolds.
- Background on Morse theory. Emphasis on the parts of the theory which generalise to Floer cohomology.
- Explicit examples (at least the two-torus).
- Arnol'd's conjecture.

This would be suitable for a master's student or beginning PhD student. Possible references would be the same as those for the first talk, as well as

- Chapter 1 of <https://people.math.ethz.ch/~salamon/PREPRINTS/floer.pdf>
- Milnor's book 'Morse theory.'
- Michael Hutching's lecture notes 'Morse homology (with an eye towards Floer theory and pseudoholomorphic curves).'
- Alex Ritter's lecture notes: <http://people.maths.ox.ac.uk/ritter/morse-cambridge.html>

(3) Lecture 3:

- Introduction to Hamiltonian Floer theory.
- Product structure in Hamiltonian Floer theory via pair-of-pants.
- Explain why a choice of Hamiltonian matters for Liouville manifolds. Which class of Hamiltonians do we want to allow?
- Symplectic cohomology via quadratically growing Hamiltonians.
- Example: Symplectic cohomology for punctured surfaces.

This would be suitable for a master's student or beginning PhD student. Possible references would be the same as those for the first talk, as well as

- Chapter 4 of <https://people.math.ethz.ch/~salamon/PREPRINTS/floer.pdf> is an excellent resource.
- Seidel's biased view: <https://arxiv.org/pdf/0704.2055.pdf>
- Chapter 1 of Abouzaid's notes: <https://arxiv.org/pdf/1312.3354.pdf>

Warning: The conventions in Salamon's notes differ from Abouzaid and Seidel's work. Namely, Salamon constructs Floer *homology*.

(4) Lecture 4

- Symplectic cohomology as a limit. Continuation maps.
- Algebraic structures on symplectic cohomology: BV structures, pair-of-pants products.

This talk would be suitable for a PhD student, or enthusiastic master's students. I would suggest Chapter 2 of Abouzaid's notes from the previous talk as a good reference. For more

details, one can read Ritter's paper <https://arxiv.org/pdf/1003.1781.pdf>, which establishes the TQFT structure on symplectic cohomology.

(5) Lecture 5

- Introduction to string topology.
- Chas-Sullivan product.
- Morse theoretical model for the homology of the free loop space.

This would be appropriate for a PhD student or enthusiastic master's student. Possible references include

- Chapter 3 of Abouzaid's lecture notes.
- Chapter 1 of Cohen-Voronov's lecture notes: <https://arxiv.org/pdf/math/0503625.pdf>.

(6) Lecture 6

- Symplectic cohomology for cotangent bundles. This is the most important example.
- Viterbo's theorem. Constructing the isomorphism to $H_*(\mathcal{L}M)$, at least heuristically.

This would be appropriate for a PhD student or enthusiastic master's student. Possible reference include

- Chapters 4-6 of Abouzaid's lecture notes.

(7) Lecture 7

- Rabinowitz Floer homology.
- Poincaré duality in this context.
- How does this relate to other examples of Poincaré duality we already understand?
- Long exact sequence for symplectic homology/ cohomology with Rabinowitz Floer homology.

This would be appropriate for a PhD student or enthusiastic master's student. Possible reference include

- Section 3 of <https://arxiv.org/pdf/0903.0768.pdf>
- The main theorems of <https://arxiv.org/pdf/2008.13161.pdf>

(8) Lecture 8

- Explain Goodwillie's theorem $H_*(\mathcal{L}M) \simeq HH_*(C_*(\Omega M))$.
- Explain Poincaré duality in this context.
- Passing this through Viterbo's theorem, what does Poincaré duality look like on $SH^*(T^*M)$?