

# Seminar: Higher algebraic K-theory

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**Meeting time:** Wednesday 10:15-11:45

**Location:** SR 0.007

**Description:** The plan is to study various approaches to higher algebraic  $K$ -theory beginning with the foundational work of Quillen. We will use original references accompanied by the book [14].

## Talks:

1. (Oct 11) Introduction and survey of pre-Quillen history [13, 14].
2. (Oct 18) Explain Quillen's definition of the higher  $K$ -groups via the Plus-construction [8, 14].
3. (Oct 25)  $K$ -theory of finite fields: Sketch the contents of [8, 14].
4. (Nov 8) Quillen's Theorem A and B [9, §1], optionally discuss  $\infty$ -categorical version from [7].
5. (Nov 15) Additivity [9, §2,§3].
6. (Nov 22) Devissage and Localization [9, §4,§5].
7. (Nov 29)  $\Gamma$ -spaces and Segal delooping [11].
8. (Dec 13) Waldhausen  $K$ -theory: Explain  $S_\bullet$ -construction and additivity [12], possibly also for stable  $\infty$ -categories [6].
9. (Dec 20)  $K$ -theory of spaces: [12], also explain  $\infty$ -categorical interpretation [6].
10. (Jan 10) Variants of  $K$ -theory: Grothendieck-Witt groups, Hermitian  $K$ -theory, see [10] and references.
11. (Jan 17) Real algebraic  $K$ -theory: [5].
12. (Jan 24) Cyclic homology and  $K$ -theory: Survey the paper [1].
13. (Jan 31) Goodwillie calculus: Sketch Goodwillie's calculus of functors and its application to the  $K$ -theory of spaces [3, 2, 4].

## References

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- [5] Lars Hesselholt and Ib Madsen. Real algebraic K-theory. *to appear*, 2013.
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- [8] D. Quillen. On the cohomology and K-theory of the general linear groups over a finite field. *Annals of Mathematics*, pages 552–586, 1972.
- [9] D. Quillen. Higher algebraic K-theory. I. In *Algebraic K-theory, I: Higher K-theories (Proc. Conf., Battelle Memorial Inst., Seattle, Wash., 1972)*, volume 341, pages 85–147. Springer, 1973.
- [10] M. Schlichting. Hermitian K-theory of exact categories. *Journal of K-theory*, 5(1):105–165, 2010.
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- [12] F. Waldhausen. Algebraic K-theory of spaces. *Algebraic and geometric topology (New Brunswick, NJ, 1983)*, 1126:318–419, 1985.
- [13] C. Weibel. The development of algebraic K-theory before 1980. *Algebra, K-theory, groups, and education (New York, 1997)*, 243:211–238, 1999.
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