Universität Hamburg
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Rational Homotopy Theory (Master)
Winter Semester 2020/21

Sheet 13

Question 13.1

Show that a nilpotent rational space of finite type has finite-dimensional rational cohomology in each degree.

Hint: Use induction and show that for a principal fibration $Y \to X$ with fiber $K(\mathbb{Q}, i)$ if $H^i(X, \mathbb{Q})$ is a finite-dimensional for all i then the same is true for the $H^j(Y, \mathbb{Q})$.

Question 13.2

Recall the cdga $M = \mathbb{Q}\langle a, b, c \mid dc = ab\rangle$ with $|a| = |b| = |c| = 1$ from example sheet 7. M is the minimal model of the topological space Z obtained as the quotient of strictly upper triangluar 3×3-matrices by strictly upper triangular 3×3-matrices with integer entries.

Compare the homotopy groups of M and Z.

Describe a factorization of the minimal model and the corresponding tower of principal fibrations as in Example 9.7.

Question 13.3

For the following topological spaces, determine their minimal model and the homotopy groups of the minimal model.

1. $S^5 \times S^2$,
2. $\mathbb{R}P^3$,
3. $\mathbb{C}P^2 \times S^3$,
4. $\mathbb{C}P^\infty \vee \mathbb{C}P^\infty$.

* You should have found the same homotopy groups in examples 1 and 3. Do the spaces also have the same homotopy groups? Can you give a topological reason why?

These questions will be discussed in the exercise class on 15.2.20.

Questions with an asterisk are more challenging.