- Basic idea : $M \subseteq M[G]$

- Forcing relation
  - $\Vdash$ - semantic (intuitive)
  - $\Vdash^*$ - syntactic (formal)

  * Proof of equivalence is involved $\Rightarrow$ you can first skip.

- Forcing theorem : $M[G] \models \phi \Rightarrow \exists p \in \mathbb{P}^M (p \Vdash \phi)$

  * First skip $M[G] \models ZFC$

- Application of forcing : " adding κ-many new reals (functions $f \in 2^\omega$) to M "

Being def. in M means there is a $\Phi_\varphi(p, \tau_1 .. \tau_n)$ such that

$$\boxed{\Phi_\varphi^M(p, \tau_1 .. \tau_n) \iff p \Vdash_{\mathbb{P}, M} \varphi(\tau_1 \dots \tau_n)}$$

$\Rightarrow$ $\Vdash$ is a (definable) class in M.

<u>Use it</u> : $\mathbb{P} \in M$ Let $Y := \{ p \in \mathbb{P} : \underset{\text{def in } M}{p \Vdash \dots} \}$

$$X := \{ \tau \in \underset{\text{set.}}{Z} \subseteq \mathbb{P}^M : \exists p \Vdash \dots \tau \dots \}$$

Then $X, Y ..$ are $\in M$.

(because $\Vdash$ is definable and $M \models$ Compr.)

$\mathbb{P} \in M$

M transitive

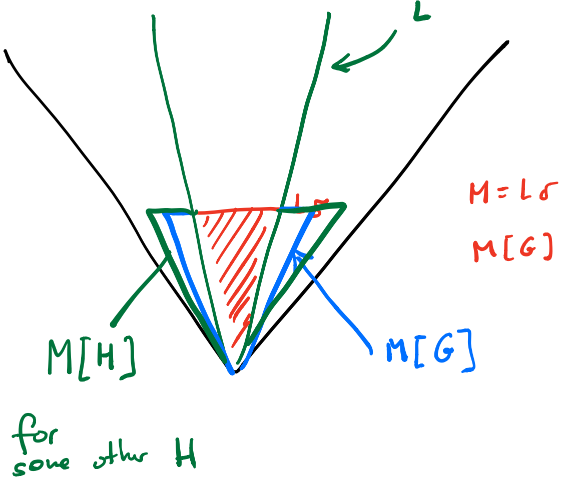
$\mathbb{P} \subseteq M$

$D \subseteq \mathbb{P}$ dense $D \in M$

$\Rightarrow D \subseteq M$

$G \subseteq \mathbb{P}$ will we $G \subseteq M$

But $G \notin M$

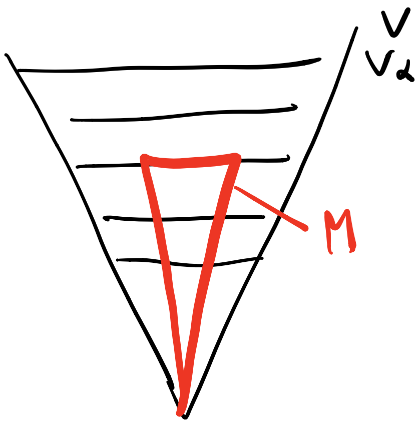


$o(M[G]) = o(M)$

- $\mathrm{Ord} \cap M$ is initial segm. of Ord

  $\beta \in \alpha \in M \rightarrow \beta \in M$

- $\mathrm{Ord} \cap M$ is itself an ordinal $=: o(M)$



$x \in M$

$rk(x) = \alpha \Rightarrow \alpha \in M$

	12:00	14:00	16:00
Wed 27	Talk 1	Talk 2	Talk 5
Thu 28	Talk 3	Talk 4	
Fr 29	Talk 6	Talk 7	

Lida + Daniel