Informal proofs

We like you to give proofs in general as informal proofs, not as formal derivations in natural deduction or Hilbert type axiom systems.

Example. Informal proof of $\neg(\varphi \vee \psi) \to \neg\varphi \wedge \neg\psi$.

Assume $\neg(\varphi \vee \psi)$. Now also assume φ. This gives $\varphi \vee \psi$, a contradiction. So, $\neg\varphi$. Similarly, assuming ψ gives a contradiction. So, $\neg\psi$. From $\neg\varphi$ and $\neg\psi$, $\neg\varphi \wedge \neg\psi$ [Note that in an informal proof we stop here.]

This is of course connected on the one hand to a formal derivation in natural deduction, on the other hand to an explanation of the validity of $\neg(\varphi \vee \psi) \to \neg\varphi \wedge \neg\psi$ according to the BHK-interpretation.

Explanation of the validity of $\neg(\varphi \vee \psi) \to \neg\varphi \wedge \neg\psi$ according to the BHK-interpretation.

We have to give a method that, given a proof of $\neg(\varphi \vee \psi)$, produces a proof of $\neg\varphi \wedge \neg\psi$.

The latter consists of a proof of $\neg\varphi$ and a proof of $\neg\psi$ plus the conclusion. So, it will suffice to give methods to produce proofs of $\neg\varphi$ and $\neg\psi$, given a proof of $\neg(\varphi \vee \psi)$. Those two methods can then be combined to a method to obtain a proof of $\neg\varphi \wedge \neg\psi$. We give the method to produce a proof of $\neg\varphi$. For $\neg\psi$ it is completely analogous. A proof of $\neg\varphi$ is a method to produce a contradiction, given a proof of φ. This means that we have to give a method that, given proofs of $\neg(\varphi \vee \psi)$ and φ produces a contradiction. A proof of φ can be transformed into a proof of $\varphi \vee \psi$ by just adding the conclusion $\varphi \vee \psi$. Combining this with the proof of $\neg(\varphi \vee \psi)$, which is a method to obtain a contradiction, given a proof of $\varphi \vee \psi$, this makes for a method that, given proofs of $\neg(\varphi \vee \psi)$ and φ, produces a contradiction, which is what we needed.