Homework 2, due Friday 15 February, before 15.00

1. Give Kripke counter-models to:

   (a) $\neg(p \wedge q) \to \neg p \vee \neg q$ [2pts]

   (b) $\neg(p \to q) \to p \wedge \neg q$ [2pts]

   (c) $[((p \to q) \to q) \wedge ((q \to p) \to p)] \to (p \vee q)$ [2pts]

2. Exercise 4 of the syllabus, on p 16:

   Prove that persistency transfers to formulas (i.e., if $w \models \phi$ and wRv then $v \models \phi$, for all propositional formulas ϕ). [4pts]

3. Show directly, **without** using Theorem 30 ($\vdash_{\mathbf{CPC}} \phi$ iff $\vdash_{\mathbf{IPC}} \phi^n$), that $((\varphi \to \chi) \wedge (\psi \to \chi) \to (\varphi \vee \psi \to \chi))^n$ is provable in **IPC**.

   You **are** allowed to use the following fact: $\vdash_{\mathbf{IPC}} \varphi^n \leftrightarrow \neg\neg\varphi^n$. [4pts]

4.* **Definition:**

   - φ is **negative** iff there is some ψ such that $\vdash_{\mathbf{IPC}} \varphi \leftrightarrow \neg\psi$
   - φ has the **down property** iff for each w which is not an end-point, if for all x with wRx and $w \neq x$ we have $x \models \varphi$, then $w \models \varphi$.

   Show that φ is negative iff it has the down property. [4pts]