Exercises in Algebra (master): Homological Algebra

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Exercise sheet no 3 for the exercise class on the 28th of April 2021

1 (Injective abelian groups)

- Prove that an abelian group A is injective if and only if it is divisible, *i.e.*, for all $0 \neq n \in \mathbb{Z}$ the map $\cdot n \colon A \to A$ is surjective.
- Let A be a finitely generated abelian group. Construct an injective abelian group I together with a monomorphism $i \colon A \to I$. (You know that such injective modules together with such monomorphisms exist in broader generality. Find a small I.)

2 (Kernels and friends) Let $\mathcal{C}$ be an abelian category and let $f \in \mathcal{C}(A, B)$.

(1) Show that
  - the kernel of f is a monomorphism and
  - the cokernel of f is an epimorphism.

(2) Prove that the image of f is a kernel of the cokernel of f.

3 ((Co)Products in poset categories) Let $(X, \leqslant)$ be a poset and consider the associated category.

- What is an initial object in $(X, \leqslant)$ and what is a terminal object in $(X, \leqslant)$?
- What is a coproduct or product in $(X, \leqslant)$? Find examples where these do not exist.