

Exercises in Algebraic Topology (master)

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Summer term 2024

Exercise sheet no 8

due: 31st of May 2024, 11:45h in H2

1 (Products of Moore spaces) (4 + 1 points)

- (1) Let A and B be arbitrary finitely generated abelian groups. Determine the homology groups of the product of the Moore spaces $M(A, n)$ and $M(B, m)$, $H_*(M(A, n) \times M(B, m))$, for arbitrary natural numbers $n, m \geq 1$.
- (2) In the special case of $M(\mathbb{Z}/2\mathbb{Z}, 1) = \mathbb{R}P^2$, what is $H_*(\mathbb{R}P^2 \times \mathbb{R}P^2)$?

2 (Splittings) (3 + 2 points)

- a) Give an explicit example for the fact that the splitting in the topological Künneth formula is not natural.
- b) Let C_* be a free chain complex and let G be an abelian group. Show that the sequence

$$0 \rightarrow H_n(C_*) \otimes G \rightarrow H_n(C_* \otimes G) \rightarrow \text{Tor}(H_{n-1}(C_*), G) \rightarrow 0$$

splits.

3 (Ext) (2 + 2 points)

For a free resolution $0 \rightarrow R \xrightarrow{i} F \xrightarrow{p} A \rightarrow 0$ and an abelian group B the cokernel of the map

$$\text{Hom}(i, B): \text{Hom}(F, B) \rightarrow \text{Hom}(R, B)$$

is $\text{Ext}(A, B)$ and hence the sequence

$$0 \rightarrow \text{Hom}(A, B) \xrightarrow{\text{Hom}(p, B)} \text{Hom}(F, B) \xrightarrow{\text{Hom}(i, B)} \text{Hom}(R, B) \rightarrow \text{Ext}(A, B) \rightarrow 0$$

is exact.

- 1) Determine $\text{Ext}(A, B)$ if A is a finitely generated abelian group.
- 2) For natural numbers n and m give an explicit formula for $\text{Ext}(\mathbb{Z}/n\mathbb{Z}, \mathbb{Z}/m\mathbb{Z})$.