Logicism - Rudolf Carnap

Edgar Andrade

Philosophy of Mathematics

ILLC - Master of Logic

Rudolf Carnap (1891-1970)

- Born on May 18, 1891, in Ronsdorf, Germany.
- A leading member of the Vienna Circle; In 1929, with Hans Hahn and Otto Neurath, he wrote the manifesto of the Circle.
- In 1935, with the aid of the American philosophers Charles Morris and Willard Van Orman Quine, Carnap moved to the United States. He became an American citizen in 1941.
- Died on September 14, 1970, at Santa Monica, California.

Motivation

Main philosophical questions:

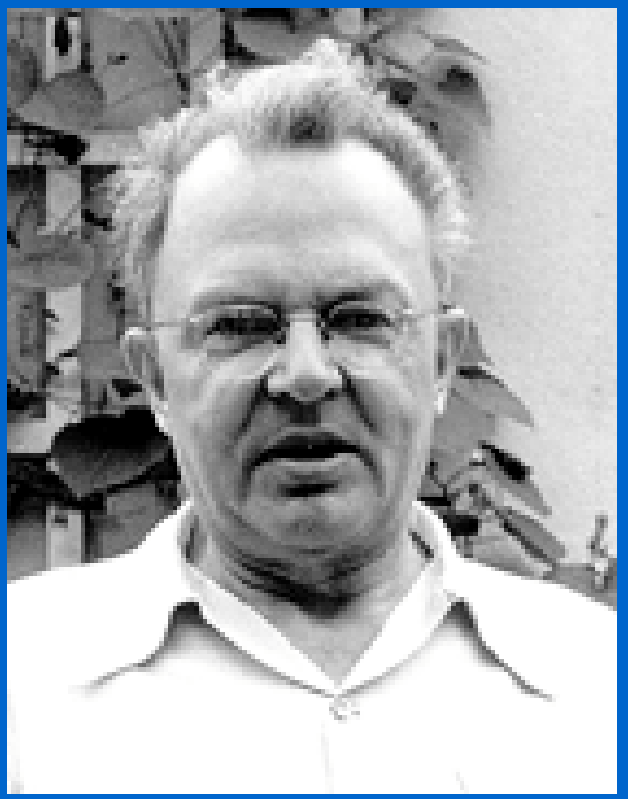
- Metaphysical questions:
  - Does the world exist?
  - Do mathematical objects exist?
- Epistemological questions:
  - How can we know the world?
  - How can we know mathematics?

Motivation (2)

- What do these questions mean?
- Linguistic turn (Dummett): Our knowledge is bounded by our language.
- Distrust on the accuracy of the natural language.
- Need for a more accurate language

Linguistic Frameworks

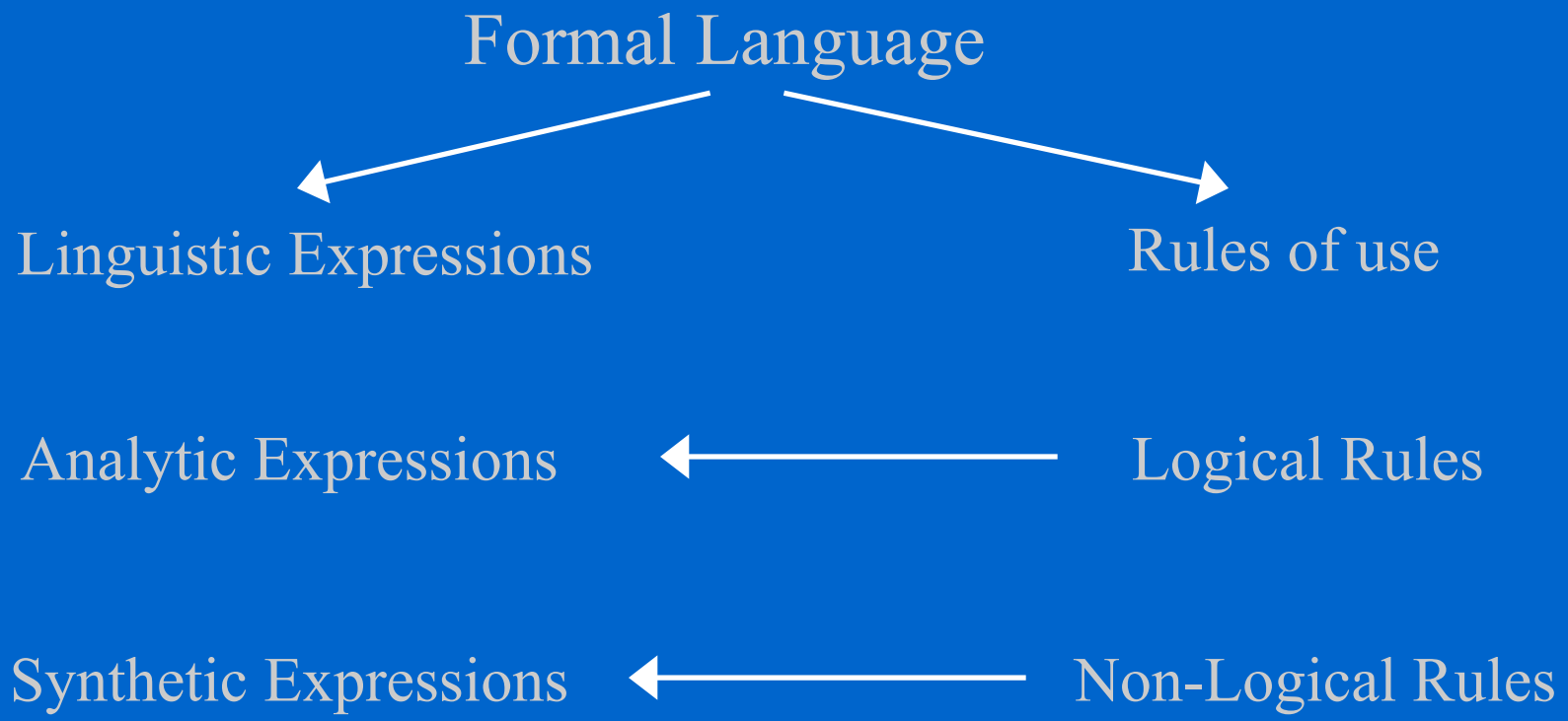
- Philosophy is the logic of Science.
- To construct a science means to construct a system of propositions.
- Philosophy is the logical analysis of these systems.



Linguistic Frameworks (2)

`A ``linguistic framework'' is an attempt formally to delineate a part of discourse. The framework should contain a precise grammar, indicating which expressions are legitimate sentences in the framework, and it should contain rules for the use of the sentences. Some of the rules may be empirical, indicating, for example, that one can assert such and such a sentence when one has a certain kind of experience. Other rules will be logical, indicating what inferences are allowed and which sentences can be asserted no matter what experience one has.'(Shapiro 2000: p.126-7)

Linguistic Frameworks (3)



Analytic Synthetic Distinction

- Let φ be a linguistic expression:
  - φ is analytic iff φ is a logical tautology.
  - φ is contradictory iff ¬φ is a logical tautology.
  - φ is synthetic iff φ is not analytic nor contradictory.

Philosophical problems

Connotative mode $\Longrightarrow$ **Formal mode**

- The concept ``red'' signifies an ultimate quality.
- The moon is a thing.
- The sum of 3 and 2 is not a thing but a number.

- The word ``red'' is an undefined fundamental symbol of language.
- ``Moon'' is the designation of a thing.
- ``3+2'' is not a designation of a thing but a designation of a number.

Philosophical problems (2)

- `Since now the data of every logical analysis can be translated in the formal mode of expression, all the questions and theorems of philosophy consequently find their place in the formal structure theory of language, that is, in the realm which we have called the Syntax of the language of Science'.(Carnap 1934: p.14)

Internal and external questions

- Let Q be a question of existence.
- Formal mode : $\exists x \varphi(x)$ (Translation of Q)
  - Internal question
    - Logical analysis
    - Empirical verification
- Informal mode:
  - External question
  - ?

Internal questions

- Q1: 'Is there a white piece of paper on my desk?'
- Q2:'Is there a prime number greater than one hundred?'

- $\Rightarrow$ Find a framework and the respective formal mode of presentation in it.

- Consider the *world of things framework*:
  - Q1: Determined by empirical verification
  - Q2: Determined by logical analysis (if the numbers are part of this framework).
- Consider the *numbers framework:*
  - Q1: Doesn't make any sense.
  - Q2: Determined by logical analysis.

External questions

- Note: we don't use a linguistic framework as reference. Therefore the questions are in the connotative mode.
- 'The connotative mode of expression is more customary and obvious; but one must use it with great care, it frequently begets muddles and pseudo-problems.' (Carnap 1934: p. 13)
- 'An external question is of a problematic character which is in need of closer examination.' (Carnap 1950: p.234)
- 'To recognize something as a real thing or event means to succeed in incorporating it into the system of things at a particular space-time position so that it fits together with the other things recognized as real, according to the rules of the framework.' (Carnap 1950: p.235)
- 'To be real in the scientific sense means to be an element of the system.' (Carnap 1950: 235)

External questions (2)

- The question of existence outside a framework doesn't have any meaning according to the definition of existence given by Carnap. It is natural that he concludes that:
- 'Unfortunately, these philosophers have so far not given a formulation of their question in terms of the common scientific language. Therefore our judgement must be that they have not succeeded in giving to the external question and to the possible answers any cognitive content.'
  (Carnap 1950: p. 237)
- 'The external questions of the reality of physical space and physical time are pseudo-questions.'(Carnap 1950: 240)

External questions (3)

- ‘To be sure, we have to face at this point an important question; but it is a practical, not a theoretical question; it is the question of whether or not to accept the new linguistic forms. The acceptance cannot be judged as being either true or false because it is not an assertion. It can only be judged as being more or less expedient, fruitful, conducive to the aim for which the language is intended.’(Carnap 1950: pp.241-2)

Nominalism?

- Influenced by the ideas of Ludwig Wittgenstein, the Circle rejected both the thesis of the reality of the external world and the thesis of its irreality as pseudo-statements; the same was the case for both the thesis of the reality of universals (abstract entities, in our present terminology) and the nominalistic thesis that they are not real and that their alleged names are not names of anything but merely *flatus vocis*. (It is obvious that the apparent negation of a pseudo-statement must also be a pseudo-statement).'(Carnap 1950: p. 242)

Discussion

- Impredicative definitions (ID):
  - Russell struggled with Frege's reduction of natural numbers because of ID. To avoid ID he came up with several ad hoc principles.
  - Carnap simply said yes to ID on pragmatic grounds.
- Mathematical necessity:
  - Mill hold that mathematical propositions aren't necessary.
  - The logical positivists hold that mathematical propositions don't have factual content and therefore are necessary.
  - By the same token, the mathematical propositions aren't synthetic, and thus they seem to be against kant (however, *they have different definitions of analytic and synthetic propositions*).

Discussion (2)

- Philosophy-first or last-if-at-all?
  - Carnap philosophical position is naturalism, or at least a sort of or a precursor of naturalism. This is very close (if different) from philosophy last-if-at-all.
- Realism in Ontology?
  - Frege: Numbers are real entities (but see Putnam & Benacerraf 1983).
  - Russell: Numbers are of the same kind as properties, e.g. ‘red’.
  - Carnap: Why are you discussing at all? The question is not a scientific question! More seriously: Mathematics doesn’t have any factual content.
- Realism in truth value?
  - Yes

Flaws of Logical Positivism

- `Against this the following objection, which on first appearance seems indeed destructive, has been repeatedly raised: -``If every proposition which does not belong either to mathematics or to the empirical investigation of facts, us meaningless, how does it fare then with your own propositions? You positivists and antimetaphysicians yourselves cut off the branch on which you sit".'(Carnap 1934: p.7)
- The thesis that ‘every statament is either analytic or verifiabe through experience’ is not analytic nor verifiable through experience. Besides, what is the ontological status of the linguistic frameworks?
- Quine: the distinction analytic-synthetic is problematic and the concept of meaning stand in need of explanation.

Flaws of Logical Positivism (2)

- The problem of knowledge in mathematics:
  - Knowledge of the correct use of mathematical language is sufficient for knowledge of mathematical propositons. But,
  - Gödel incompleteness theorem implies reliability on relative consistency.
  - Fermat's last theorem: We understand the formulation, but cannot give a proof. (Reply: one can have the knowledge necessary to understand a given true proposition without thereby having the resources to know that it is true.)

References

- Carnap 1934. *On the character of Philosophic Problems*, Philosophy of Science, Vol. 1, No. 1 January: 5-19.
- Carnap 1950. *Empiricism, Semantics, and Ontology*, reprinted in Benacerraf & Putnam, 1983: 241-57.
- Shapiro 2000. *Thinking about mathematics*, Oxford University Press.