Core Logic

1st Semester 2006/2007, period a & b

Dr Benedikt Löwe

Origins of Logic

- Greek mathematics
- Rhetoric: “Eristic” and “Sophistry”

Greek mathematics.

- Pre-greek mathematics was not primarily concerned with proof, but more with computation. (Egyptians, Babylonians)
  Geometry = measurement of the earth
- Thales of Miletus (c.625-c.546 BC): the first proof
  Dmitri **Panchenko**, Thales and the Origin of Theoretical Reasoning, **Configurations** 1 (1993), p. 387-414
- Pythagoras (c.569-c.475 BC)
- Mathematics built on proof:
  - Theaetetus (c.417-c.369 BC); student of Socrates
  - Euclid (c.325-c.265 BC); compilation of mathematical knowledge

Mathematical techniques.

- Proof by contradiction

Claim. $\sqrt{2}$ is not a fraction of integers.

Suppose it were, then there are integers n and m without common divisor such that

$$\sqrt{2} = \frac{n}{m}.$$

But then

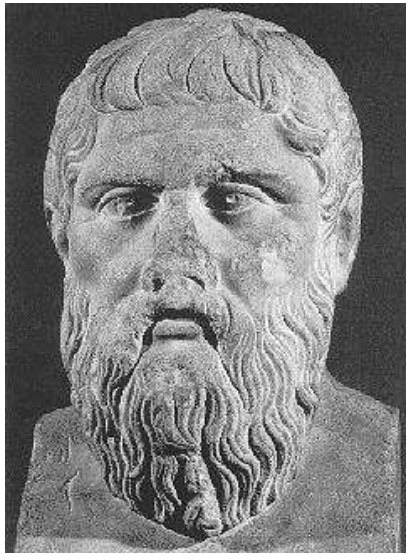
$$2m^2 = n^2.$$

In particular, n must be even. But then n^2 must be divisible by 4, and so m must be even. Contradiction.

Informal logic.

- The Dialectic method.
  - Proof by contradiction in mathematics.
  - Zeno of Elea (c.490-c.425 BC)
  - Socrates (469-399 BC; *elenchus*, *diairesis*)
- Argumentation in everyday life
  - Sophists
  - Public disputations according to rules for questioner and answerer
  - Plato, *Euthydemus*
  - Aristotle, *Topics* and *Rhetoric*
  - Megarians (Sep 27)

Plato.



Plato (c.427-347 BC)

- Student and follower of Socrates until 399 B.C.
- *399-387 BC:* Plato travels widely, including Italy and Sicily
- *387 BC:* Plato founds the **Academy**
- *367 BC:* Plato is invited to Sicily by Dionysios II.
- *347 BC:* Plato dies and is succeeded by Speusippus

The Platonic Academy.

387 BC – 526 AD
Academia was a public garden named after its donator *Academus*.

Μηδεὶς ἀγεωμέτρητος εἰσίτω μου τὴν στέγην

David **Fowler**, The Mathematics of Plato's Academy: A New Reconstruction
John **Dillon**, The Heirs of Plato: A Study of the Old Academy (347-274 BC), Oxford, 2003

Members. Speusippus (347-339), Xenocrates (339-314), Polemo (314-276), Crates, Crantor, Arcesilaus (268-240), Lacydes, Evander, Hegesinus, Carneades, Clitomachus, and Philo ... and **Aristotle**.

Theoria et Praxis (1).

The School of Athens (Raffaello Sanzio; 1509)

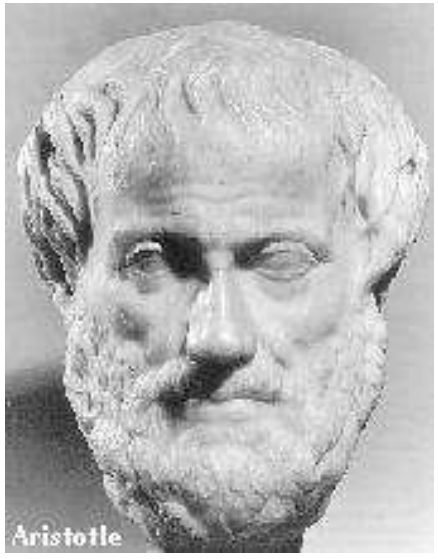
Theoria et Praxis (2).

[Uestium Philosophiae] in extremo margine Π *Graecum, in supremo uero* Θ *legebatur intextum atque inter utrasque litteras in scalarum modum gradus quidam insigniti uidebantur, quibus ab inferiore ad superius elementum esset ascensus.*

Boëthius, *Consolatio Philosophiae*
Book 1, Prosa 1

On the lowest border of [the garments of*Philosophia*] a Greek Π was embroidered, while on the highest a Θ could be read, and between both letters an ascent could be seen in the manner of stairs, by which you could move from the lower to the higher element.

Aristotle.



Aristotle (384-322 BC)

- *367 BC*: Aristotle joins the Academy.
- *347 BC*: Plato dies, Aristotle leaves Athens.
- *343-336 BC*: Aristotle works at the court of Macedonia.
- *335 BC*: Aristotle founds the *Lyceum* in Athens (Peripatetics).
- *323 BC*: Alexander the Great dies, Aristotle retires to Chalcis.

Esoteric / exoteric.

Aristotle:

- **Esoteric works**: lecture notes and textbooks, designed for use within the Lyceum.
- **Exoteric works**: dialogues (modelled after the Platonic dialogues), designed for the general public.

"Plato's unwritten doctrine":

- Neoplatonism: Plotinus (204-270 AD)
- Porphyry (c.232-c.305 AD)
- [St. Augustine (354-430 AD)]
- Proclus (411-485 AD)

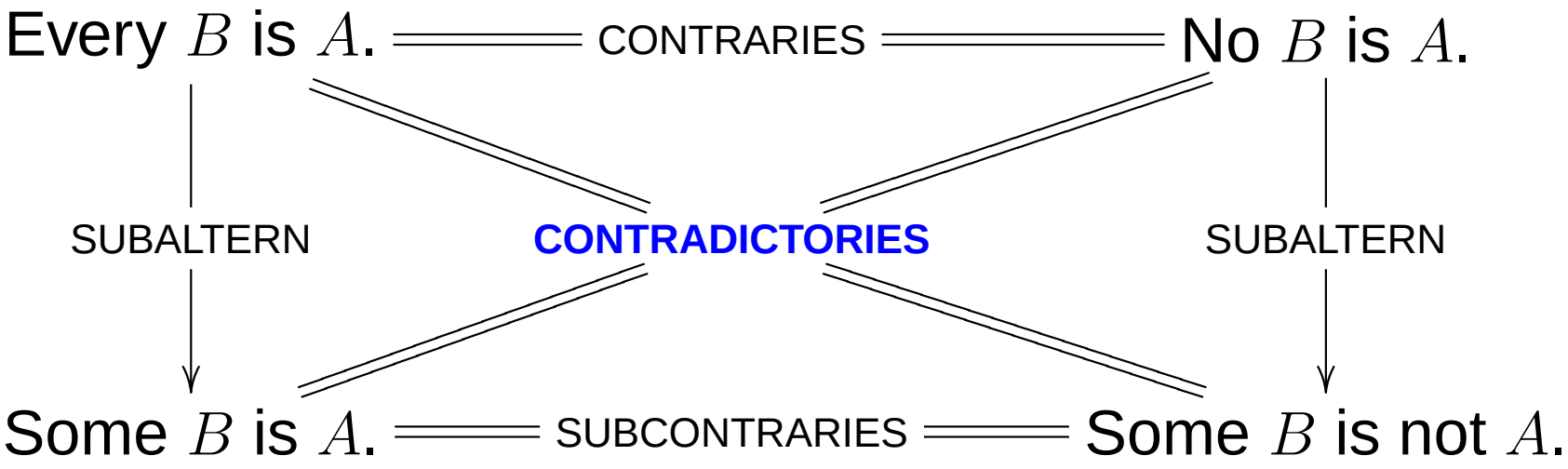
Aristotle's work on logic.

The **Organon**.

- **Categories**: Classification of types of predicates
- **On Interpretation***(De interpretatione): Basics of philosophy of language, subject-predicate distinction, Square of Oppositions*
- **Prior Analytics**: Syllogistics
- **Posterior Analytics**: More on syllogistics
- **Topics**: Logic except for syllogistics
- **On Sophistical Refutations** *(De Sophisticis Elenchis)*: Fallacies

The square of oppositions.

Aristotle, *De interpretatione*



- **Contradictory** propositions cannot both be true and they cannot both be false.
- **Contrary** propositions cannot both be true but can both be false.
- **Subcontrary** propositions cannot both be false but can both be true.
- A **subaltern** must be true if its **superaltern** is true, and the **superaltern** must be false if the **subaltern** is false.