# Core Logic

## *1st Semester 2006/2007, period a & b*

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# Aristotle's work on logic.

The **Organon**.

- **Categories**
- **On Interpretation**
- **Prior Analytics**
- **Posterior Analytics**
- **Topics**
- **On Sophistical Refutations** *(De Sophisticis Elenchis)*

# The Categories.

**Aristotle**, *Categories*:
**The ten categories** (1b25).

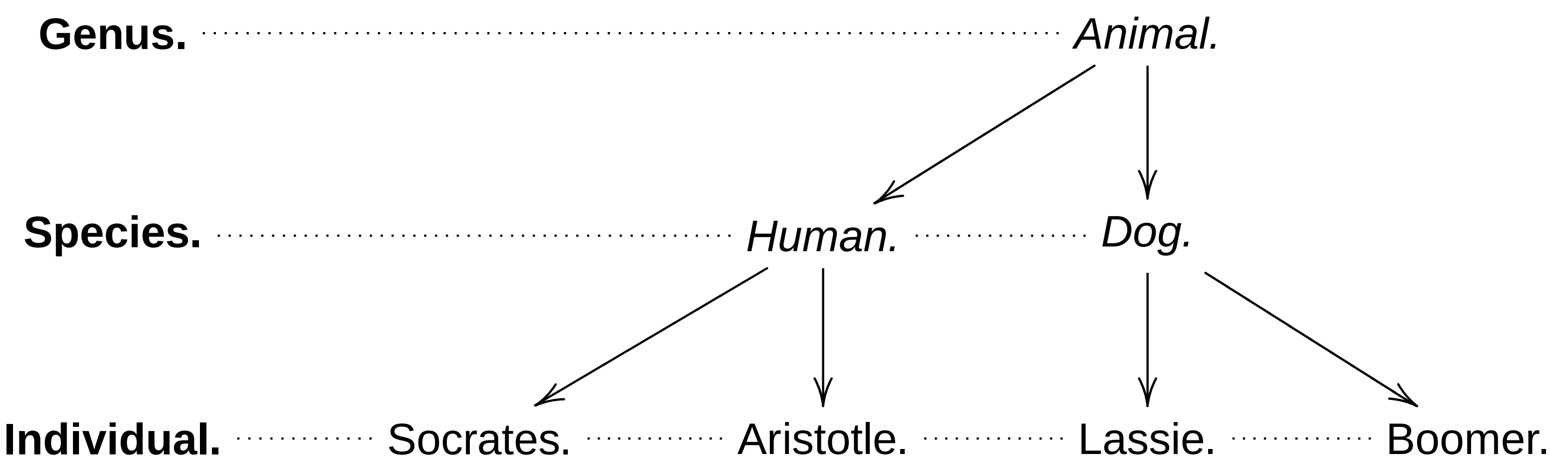
| | |
|---|---|
| Substance | When |
| Quality | Position |
| Quantity | Having |
| Relation | Action |
| Where | Passion |

**The two ways of predication**.

- *essential predication*: “Socrates is a human being”; “*human* IS SAID OF Socrates” Grice: IZZing
- *accidental predication*: “Socrates is wise”; “*wisdom* IS IN Socrates” Grice: HAZZing

# Essential predication.

- “essential”: You cannot deny the predicate without changing the meaning of the subject.
  - “*animal* IS SAID OF *human*”.
  - “*human* IS SAID OF Socrates”.
- IS SAID OF is a transitive relation.
- Related to the category tree:



# Substances.

| Universal substances<br>*human*, *animal* | Universal accidents<br>*wisdom* |
|---|---|
| **Particular substances**<br>Socrates, Aristotle | **Particular accidents** |

- **Plato** (↑). The **universal substances** are the (only) real things.
- **Aristotle** (↓). Without the **particular substances**, nothing would exist.

# Matter & Form.

- *Categories* / *De anima*: There are three kinds of substance: *matter*, *form* and the compound of the two.
- *Matter* is *potentiality*; *form* is *actuality*.
- **Aristotle in the *Metaphysics*:** *Primary substances* cannot be compounds, not even of matter and form. Matter cannot be *primary*, therefore, the *primary substance* is the form.
- *Metaphysics Z* (1037a6): "it is also clear that the soul is the primary substance, the body is matter, and man or animal is composed of the two as universal."

The intricate connection between the analysis of predication (philosophy of language, semantics, logic) and the analysis of 'the soul' will become important in the medieval context.

# The most famous syllogism.

Every man is mortal.
**Socrates** is a man.

---

**Socrates** is mortal.

Proper name / “Particular substance”

# A more typical syllogism.

Every animal is mortal.
Every man is an animal.

---

Every man is mortal.

Every $B$ is an $A$.
Every $C$ is a $B$.

---

Every $C$ is an $A$.

**“Barbara”**

“a valid mood”
mood = *modus*

# Another valid mood.

Every philosopher is mortal.
Some teacher is a philosopher.

---

Some teacher is mortal.

Every $B$ is an $A$.
Some $C$ is a $B$.

---

Some $C$ is an $A$.

**“Darii”**

# A similar but invalid mood.

| **"Darii"** | |
|---|---|
| Every $B$ is an $A$. | Every $A$ is a $B$. |
| Some $C$ is a $B$. | Some $C$ is a $B$. |
| Some $C$ is an $A$. | Some $C$ is an $A$. |

Every philosopher is mortal.
Some nonphilosopher is mortal.

~~Some nonphilosopher is a philosopher.~~

# Yet another very similar mood.

| "Darii" | The invalid mood | "Datisi" |
|---|---|---|
| Every $B$ is an $A$. | Every $A$ is a $B$. | Every $B$ is a $A$. |
| Some $C$ is a $B$. | Some $C$ is a $B$. | Some $B$ is a $C$. |
| Some $C$ is an $A$. | Some $C$ is an $A$. | Some $C$ is an $A$. |

"Some $C$ is a $B$" and "Some $B$ is a $C$" are intuitively equivalent.

"Every $B$ is an $A$" and "Every $A$ is a $B$" aren't.

# A first conversion rule.

This yields a simple formal (syntactical) conversion rule:

“Some $X$ is a $Y$”

*can be converted to*

“Some $Y$ is an $X$.”

This rule is validity-preserving and syntactical.

# Back to *Darii* and *Datisi*.

**"Darii"**

Every $B$ is an $A$.
Some $C$ is a $B$.

---

Some $C$ is an $A$.

**"Datisi"**

Every $B$ is a $A$.
Some $B$ is a $C$.

---

Some $C$ is an $A$.

## Simple Conversion

"Some $X$ is a $Y$" $\rightsquigarrow$ "Some $Y$ is an $X$"

# Methodology of Syllogistics.

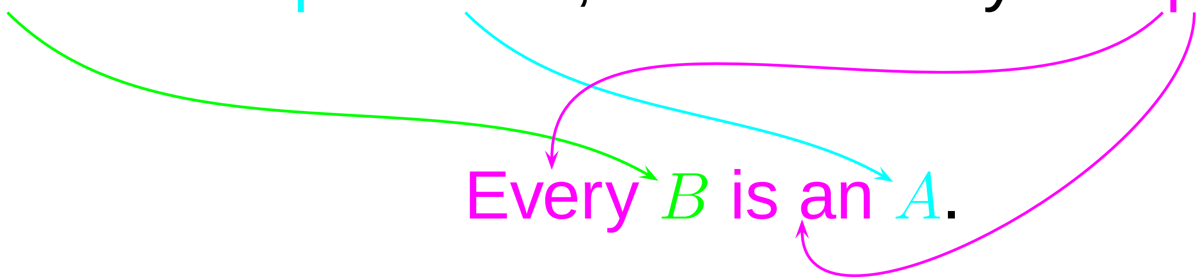
- Start with a list of obviously valid moods (perfect syllogisms $\cong$ “axioms”)...
- ...and a list of conversion rules,
- derive all valid moods from the perfect syllogisms by conversions,
- and find counterexamples for all other moods.

# Notation (1).

Syllogistics is a term logic, not propositional or predicate logic.

We use capital letters $A$, $B$, and $C$ for terms, and sometimes $X$ and $Y$ for variables for terms.

Terms (*termini*) form part of a categorical proposition. Each categorical proposition has two terms:
a subject and a predicate, connected by a copula.



# Notation (2).

There are four copulae:

- The universal affirmative: Every — is a —. a
- The universal negative: No — is a —. e
- The particular affirmative: Some — is a —. i
- The particular negative: Some — is not a —. o

Every $B$ is an $A$. $\rightsquigarrow$ $A$a$B$
No $B$ is an $A$. $\rightsquigarrow$ $A$e$B$
Some $B$ is an $A$. $\rightsquigarrow$ $A$i$B$
Some $B$ is not an $A$. $\rightsquigarrow$ $A$o$B$

**Contradictories:** a–o & e–i.

# Notation (3).

| | | |
|---|---|---|
| | Every $B$ is an $A$ | $A$a $B$ |
| **Barbara** | Every $C$ is a $B$ | $B$a $C$ |
| | Every $C$ is an $A$ | $A$a $C$ |

Each syllogism contains three terms and three categorial propositions. Each of its categorial propositions contains two of its terms. Two of the categorial propositions are premises, the other is the conclusion.
The term which is the predicate in the conclusion, is called the major term, the subject of the conclusion is called the minor term, the term that doesn't occur in the conclusion is called the middle term.

# Notation (4).

**Barbara**

| | |
|---|---|
| Every $B$ is an $A$ | $A$ a $B$ |
| Every $C$ is a $B$ | $B$ a $C$ |
| Every $C$ is an $A$ | $A$ a $C$ |

Major term / Minor term / Middle term

Only one of the premises contains the major term. This one is called the major premise, the other one the minor premise.

Ist Figure

$A$ — $B$, $B$ — $C$ : $A$ — $C$

IInd Figure

$B$ — $A$, $B$ — $C$ : $A$ — $C$

IIIrd Figure

$A$ — $B$, $C$ — $B$ : $A$ — $C$

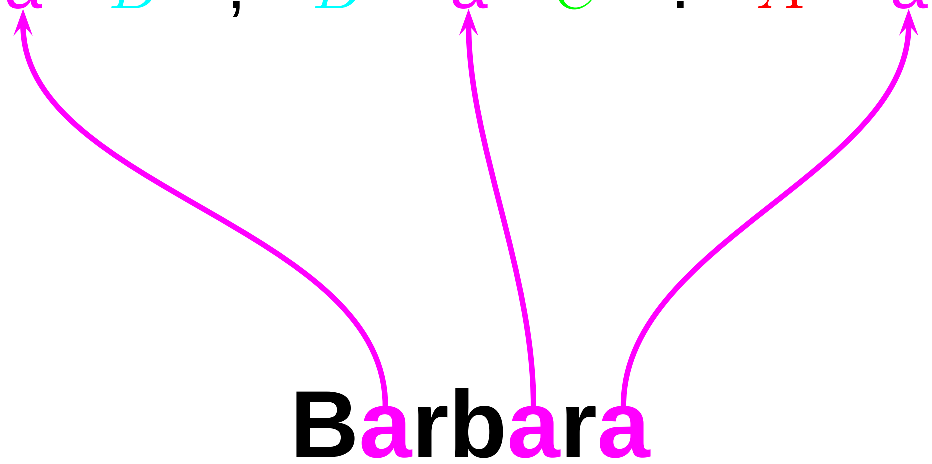
IVth Figure

$B$ — $A$, $C$ — $B$ : $A$ — $C$

# Notation (5).

If you take a figure, and insert three copulae, you get a mood.

Ist Figure: $A$ a $B$ , $B$ a $C$ : $A$ a $C$



# Combinatorics of moods.

With four copulae and three slots, we get

$$4^3 = 64$$

moods from each figure, *i.e.*, $4 \times 64 = 256$ in total.
Of these, 24 have been traditionally seen as valid.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A$ | a | $B$ | , | $B$ | i | $C$ | : | $A$ | i | $C$ | |
| D | **a** | r | | | **i** | | | | **i** | | $\rightsquigarrow$ **Darii** |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A$ | a | $B$ | , | $C$ | i | $B$ | : | $A$ | i | $C$ | |
| D | **a** | t | | | **i** | s | | | **i** | | $\rightsquigarrow$ **Datisi** |

# The 24 valid moods (1).

| | | | | | | |
|---|---|---|---|---|---|---|
| Ist figure | $A\text{a}B$ | , | $B\text{a}C$ | : | $A\text{a}C$ | **Barbara** |
| | $A\text{e}B$ | , | $B\text{a}C$ | : | $A\text{e}C$ | **Celarent** |
| | $A\text{a}B$ | , | $B\text{i}C$ | : | $A\text{i}C$ | **Darii** |
| | $A\text{e}B$ | , | $B\text{i}C$ | : | $A\text{o}C$ | **Ferio** |
| | $A\text{a}B$ | , | $B\text{a}C$ | : | $A\text{i}C$ | **Barbari** |
| | $A\text{e}B$ | , | $B\text{a}C$ | : | $A\text{o}C$ | **Celaront** |
| IInd figure | $B\text{e}A$ | , | $B\text{a}C$ | : | $A\text{e}C$ | **Cesare** |
| | $B\text{a}A$ | , | $B\text{e}C$ | : | $A\text{e}C$ | **Camestres** |
| | $B\text{e}A$ | , | $B\text{i}C$ | : | $A\text{o}C$ | **Festino** |
| | $B\text{a}A$ | , | $B\text{o}C$ | : | $A\text{o}C$ | **Baroco** |
| | $B\text{e}A$ | , | $B\text{a}C$ | : | $A\text{o}C$ | **Cesaro** |
| | $B\text{a}A$ | , | $B\text{e}C$ | : | $A\text{o}C$ | **Camestrop** |

# The 24 valid moods (2).

| | | | | | | |
|---|---|---|---|---|---|---|
| IIIrd figure | $A\mathsf{a}B$ | , | $C\mathsf{a}B$ | : | $A\mathsf{i}C$ | **Darapti** |
| | $A\mathsf{i}B$ | , | $C\mathsf{a}B$ | : | $A\mathsf{i}C$ | **Disamis** |
| | $A\mathsf{a}B$ | , | $C\mathsf{i}B$ | : | $A\mathsf{i}C$ | **Datisi** |
| | $A\mathsf{e}B$ | , | $C\mathsf{a}B$ | : | $A\mathsf{o}C$ | **Felapton** |
| | $A\mathsf{o}B$ | , | $C\mathsf{a}B$ | : | $A\mathsf{o}C$ | **Bocardo** |
| | $A\mathsf{e}B$ | , | $C\mathsf{i}B$ | : | $A\mathsf{o}C$ | **Ferison** |
| IVth figure | $B\mathsf{a}A$ | , | $C\mathsf{a}B$ | : | $A\mathsf{i}C$ | **Bramantip** |
| | $B\mathsf{a}A$ | , | $C\mathsf{e}B$ | : | $A\mathsf{e}C$ | **Camenes** |
| | $B\mathsf{i}A$ | , | $C\mathsf{a}B$ | : | $A\mathsf{i}C$ | **Dimaris** |
| | $B\mathsf{e}A$ | , | $C\mathsf{a}B$ | : | $A\mathsf{o}C$ | **Fesapo** |
| | $B\mathsf{e}A$ | , | $C\mathsf{i}B$ | : | $A\mathsf{o}C$ | **Fresison** |
| | $B\mathsf{a}A$ | , | $C\mathsf{e}B$ | : | $A\mathsf{o}C$ | **Camenop** |

# Reminder.

In syllogistics, all terms are nonempty.
**Barbari.** $A$a$B$, $B$a$C$: $A$i$C$.

Every unicorn is a white horse.
Every white horse is white.

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Some unicorn is white.

In particular, this white unicorn exists.

# The perfect moods.

> Τέλειον μὲν οὖν καλῶ συλλογισμὸν τὸν μηδενὸς ἄλλου προσδεόμενον παρὰ τὰ εἰλημμένα πρὸς τὸ φανῆναι τὸ ἀναγκαῖον. *(An.Pr. I.i)*

Aristotle discusses the first figure in *Analytica Priora* I.iv, identifies **Barbara**, **Celarent**, **Darii** and **Ferio** as *perfect* and then concludes

> Δῆλον δὲ καὶ ὅτι πάντες οἱ ἐν αὐτῷ συλλογισμοὶ τέλειοί εἰσι ... καλῶ δὲ τὸ τοιοῦτον σχῆμα πρῶτον. *(An.Pr. I.iv)*

# Axioms of Syllogistics.

So the Axioms of Syllogistics according to Aristotle are:

**Barbara.** $A$a$B$, $B$a$C$ : $A$a$C$
**Celarent.** $A$e$B$, $B$a$C$: $A$e$C$
**Darii.** $A$a$B$, $B$i$C$ : $A$i$C$
**Ferio.** $A$e$B$, $B$i$C$: $A$o$C$

# Simple and accidental conversion.

- Simple (*simpliciter*).
  - $X\mathsf{i}Y \rightsquigarrow Y\mathsf{i}X$.
  - $X\mathsf{e}Y \rightsquigarrow Y\mathsf{e}X$.
- Accidental (*per accidens*).
  - $X\mathsf{a}Y \rightsquigarrow X\mathsf{i}Y$.
  - $X\mathsf{e}Y \rightsquigarrow X\mathsf{o}Y$.

# Syllogistic proofs (1).

We use the letters $t_{ij}$ for terms and the letters $k_i$ stand for copulae. We write a mood in the form

$$\frac{\begin{array}{c} t_{11}\ k_1\ t_{12} \\ t_{21}\ k_2\ t_{22} \end{array}}{t_{31}\ k_3\ t_{32}},$$

for example,

$$\frac{\begin{array}{c} \text{AaB} \\ \text{BaC} \end{array}}{\text{AaC}}$$

for **Barbara**. We write $M_i$ for $t_{i1}\ k_i\ t_{i2}$ and define some operations on moods.

# Syllogistic proofs (2).

- For $i \in \{1,2,3\}$, the operation $\mathsf{s}_i$ can only be applied if $k_i$ is either ‘i’ or ‘e’. In that case, $\mathsf{s}_i$ interchanges $t_{i1}$ and $t_{i2}$.
- For $i \in \{1,2\}$, let $\mathsf{p}_i$ be the operation that changes $k_i$ to its subaltern (if it has one), while $\mathsf{p}_3$ is the operation that changes $k_3$ to its superaltern (if it has one).
- Let $\mathsf{m}$ be the operation that exchanges $M_1$ and $M_2$.
- For $i \in \{1,2\}$, let $\mathsf{c}_i$ be the operation that first changes $k_i$ and $k_3$ to their contradictories and then exchanges $M_i$ and $M_3$.
- Let $\mathsf{per}_\pi$ be the permutation $\pi$ of the letters A, B, and C, applied to the mood.

$$\frac{\text{BaA} \quad \text{AaC}}{\text{BaC}} \xrightarrow{\mathsf{per}} \frac{\text{AaB} \quad \text{BaC}}{\text{AaC}}$$

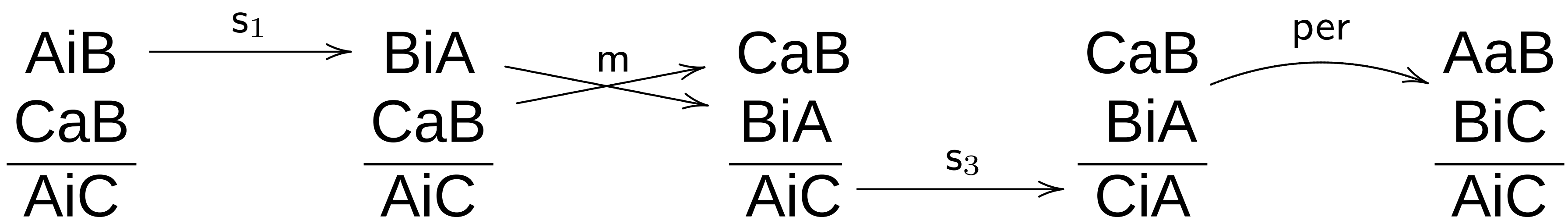
# Syllogistic proofs (3).

Given any set $\mathfrak{B}$ of “basic moods”, a $\mathfrak{B}$-**proof** of a mood $M = M_1, M_2{:}M_3$ is a sequence $\langle \mathsf{o}_1, ..., \mathsf{o}_n \rangle$ of operations such that

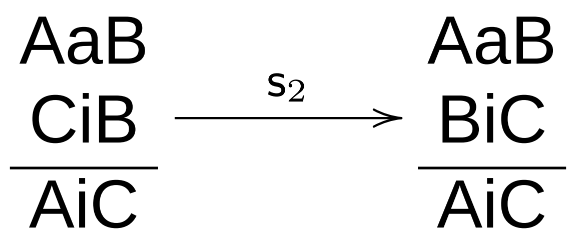
- Only $\mathsf{o}_1$ can be of the form $\mathrm{c}_1$ or $\mathrm{c}_2$ (but doesn’t have to be).
- The sequence of operations, if applied to $M$, yields an element of $\mathfrak{B}$.

# Syllogistic proofs (4).

$\langle s_1, m, s_3, per_{AC} \rangle$ is a proof of **Disamis** (from **Darii**) :

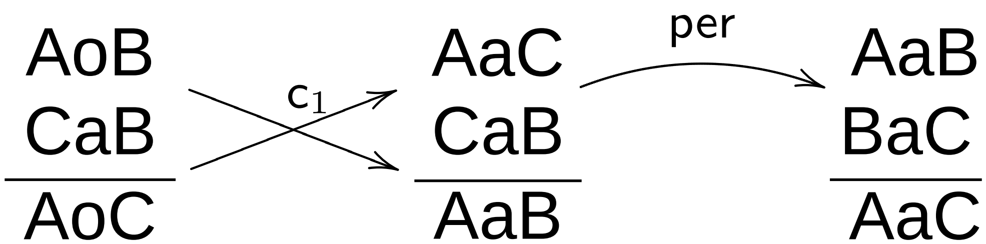



$\langle s_2 \rangle$ is a proof of **Datisi** (from **Darii**) :



# Syllogistic proofs (5).

$\langle \mathsf{c}_1, \mathsf{per}_{BC} \rangle$ is a proof of **Bocardo** by contradiction (from **Barbara**) :



# Syllogistic proofs (6).

Let $\mathfrak{B}$ be a set of moods and $M$ be a mood. We write $\mathfrak{B} \vdash M$ if there is $\mathfrak{B}$-proof of $M$.

# Mnemonics (1).

*Bárbara, Célarént, Darií, Ferióque prióris,*
*Césare, Cámestrés, Festíno, Baróco secúndae.*
*Tértia Dáraptí, Disámis, Datísi, Felápton,*
*Bocárdo, Feríson habét. Quárta ínsuper áddit*
*Brámantíp, Camenés, Dimáris, Fesápo, Fresíson.*

“These words are more full of meaning than any that were ever made.” (Augustus de Morgan)

# Mnemonics (2).

- The first letter indicates to which one of the four perfect moods the mood is to be reduced: 'B' to Barbara, 'C' to Celarent, 'D' to Darii, and 'F' to Ferio.
- The letter 's' after the $i$th vowel indicates that the corresponding proposition has to be simply converted, *i.e.*, a use of $\mathsf{s}_i$.
- The letter 'p' after the $i$th vowel indicates that the corresponding proposition has to be accidentally converted ("*per accidens*"), *i.e.*, a use of $\mathsf{p}_i$.
- The letter 'c' after the first or second vowel indicates that the mood has to be proved indirectly by proving the contradictory of the corresponding premiss, *i.e.*, a use of $\mathsf{c}_i$.
- The letter 'm' indicates that the premises have to be interchanged ("moved"), *i.e.*, a use of $\mathsf{m}$.
- All other letters have only aesthetic purposes.

# A metatheorem.

We call a proposition negative if it has either ‘e’ or ‘o’ as copula.

**Theorem** (Aristotle). If $M$ is a mood with two negative premises, then

$$\mathfrak{B}_{\mathrm{BCDF}} \nvdash M.$$

# Metaproof (1).

Suppose $\mathsf{o} := \langle \mathsf{o}_1, ..., \mathsf{o}_n \rangle$ is a $\mathfrak{B}_{\mathrm{BCDF}}$-proof of $M$.

- The $\mathsf{s}$-rules don't change the copula, so if $M$ has two negative premises, then so does $\mathsf{s}_i(M)$.
- The superaltern of a negative proposition is negative and the superaltern of a positive proposition is positive. Therefore, if $M$ has two negative premises, then so does $\mathsf{p}_i(M)$.
- The $\mathsf{m}$-rule and the $\mathsf{per}$-rules don't change the copula either, so if $M$ has two negative premises, then so do $\mathsf{m}(M)$ and $\mathsf{per}_\pi(M)$.

As a consequence, if $\mathsf{o}_1 \neq \mathsf{c}_i$, then $\mathsf{o}(M)$ has two negative premisses.We check that none of **Barbara**, **Celarent**, **Darii** and **Ferio** has two negative premisses, and are done, as $\mathsf{o}$ cannot be a proof of $M$.

# Metaproof (2).

So, $\mathsf{o}_1 = \mathsf{c}_i$ for either $i = 1$ or $i = 2$. By definition of $\mathsf{c}_i$, this means that the contradictory of one of the premisses is the conclusion of $\mathsf{o}_1(M)$. Since the premisses were negative, the conclusion of $\mathsf{o}_1(M)$ is positive. Since the other premiss of $M$ is untouched by $\mathsf{o}_1$, we have that $\mathsf{o}_1(M)$ has at least one negative premiss and a positive conclusion. The rest of the proof $\langle \mathsf{o}_2, ..., \mathsf{o}_n \rangle$ may not contain any instances of $\mathsf{c}_i$.

Note that none of the rules $\mathsf{s}$, $\mathsf{p}$, $\mathsf{m}$ and $\mathsf{per}$ change the copula of the conclusion from positive to negative.

So, $\mathsf{o}(M)$ still has at least one negative premiss and a positive conclusion. Checking **Barbara**, **Celarent**, **Darii** and **Ferio** again, we notice that none of them is of that form. Therefore, $\mathsf{o}$ is not a $\mathfrak{B}_{\mathrm{BCDF}}$-proof of $M$. Contradiction. q.e.d.

# Other metatheoretical results.

- If $M$ has two particular premises (i.e., with copulae 'i' or 'o'), then $\mathrm{BCDF} \nvdash M$ (**Exercise 10**).
- If $M$ has a positive conclusion and one negative premiss, then $\mathrm{BCDF} \nvdash M$.
- If $M$ has a negative conclusion and one positive premiss, then $\mathrm{BCDF} \nvdash M$.
- If $M$ has a universal conclusion (i.e., with copula 'a' or 'e') and one particular premiss, then $\mathrm{BCDF} \nvdash M$.