Methodology of Syllogistics.

- Start with a list of obviously valid moods (perfect syllogisms $\cong$ “axioms”)...
- ...and a list of conversion rules,
- derive all valid moods from the perfect syllogisms by conversions,
- and find counterexamples for all other moods.

Notation (1).

Syllogistics is a term logic, not propositional or predicate logic.

We use capital letters A, B, and C for terms, and sometimes X and Y for variables for terms.

Terms (*termini*) form part of a categorical proposition. Each categorical proposition has two terms:
a subject and a predicate, connected by a copula.

Every B is an A.

Notation (2).

There are four copulae:

- The universal affirmative: Every — is a —. a
- The universal negative: No — is a —. e
- The particular affirmative: Some — is a —. i
- The particular negative: Some — is not a —. o

Every B is an A. $\rightsquigarrow$ AaB
No B is an A. $\rightsquigarrow$ AeB
Some B is an A. $\rightsquigarrow$ AiB
Some B is not an A. $\rightsquigarrow$ AoB

Contradictories: a–o & e–i.

Notation (3).

	Every B is an A	Aa B
Barbara	Every C is a B	Ba C
	Every C is an A	Aa C

Each syllogism contains three terms and three categorial propositions. Each of its categorial propositions contains two of its terms. Two of the categorial propositions are premises, the other is the conclusion.
The term which is the predicate in the conclusion, is called the major term, the subject of the conclusion is called the minor term, the term that doesn't occur in the conclusion is called the middle term.

Notation (4).

	Every B is an A	A a B
Barbara	Every C is a B	B a C
	Every C is an A	A a C

Major term / Minor term / Middle term

Only one of the premises contains the major term. This one is called the major premise, the other one the minor premise.

Ist Figure

$$A — B, B — C : A — C$$

IInd Figure

$$B — A, B — C : A — C$$

IIIrd Figure

$$A — B, C — B : A — C$$

IVth Figure

$$B — A, C — B : A — C$$

Notation (5).

If you take a figure, and insert three copulae, you get a mood.

Ist Figure: A a B , B a C : A a C

Barbara

Combinatorics of moods.

With four copulae and three slots, we get

$$4^3 = 64$$

moods from each figure, *i.e.*, $4 \times 64 = 256$ in total.
Of these, 24 have been traditionally seen as valid.

A	a	B	,	B	i	C	:	A	i	C		
D	**a**	r			**i**				**i**		$\rightsquigarrow$	**Darii**
A	a	B	,	C	i	B	:	A	i	C		
D	**a**	t			**i**	s			**i**		$\rightsquigarrow$	**Datisi**

The 24 valid moods (1).

Ist figure	AaB	,	BaC	:	AaC	**Barbara**
	AeB	,	BaC	:	AeC	**Celarent**
	AaB	,	BiC	:	AiC	**Darii**
	AeB	,	BiC	:	AoC	**Ferio**
	AaB	,	BaC	:	AiC	**Barbari**
	AeB	,	BaC	:	AoC	**Celaront**
IInd figure	BeA	,	BaC	:	AeC	**Cesare**
	BaA	,	BeC	:	AeC	**Camestres**
	BeA	,	BiC	:	AoC	**Festino**
	BaA	,	BoC	:	AoC	**Baroco**
	BeA	,	BaC	:	AoC	**Cesaro**
	BaA	,	BeC	:	AoC	**Camestrop**

The 24 valid moods (2).

IIIrd figure	$A\mathsf{a}B$	,	$C\mathsf{a}B$	:	$A\mathsf{i}C$	**Darapti**
	$A\mathsf{i}B$	,	$C\mathsf{a}B$	:	$A\mathsf{i}C$	**Disamis**
	$A\mathsf{a}B$	,	$C\mathsf{i}B$	:	$A\mathsf{i}C$	**Datisi**
	$A\mathsf{e}B$	,	$C\mathsf{a}B$	:	$A\mathsf{o}C$	**Felapton**
	$A\mathsf{o}B$	,	$C\mathsf{a}B$	:	$A\mathsf{o}C$	**Bocardo**
	$A\mathsf{e}B$	,	$C\mathsf{i}B$	:	$A\mathsf{o}C$	**Ferison**
IVth figure	$B\mathsf{a}A$	,	$C\mathsf{a}B$	:	$A\mathsf{i}C$	**Bramantip**
	$B\mathsf{a}A$	,	$C\mathsf{e}B$	:	$A\mathsf{e}C$	**Camenes**
	$B\mathsf{i}A$	,	$C\mathsf{a}B$	:	$A\mathsf{i}C$	**Dimaris**
	$B\mathsf{e}A$	,	$C\mathsf{a}B$	:	$A\mathsf{o}C$	**Fesapo**
	$B\mathsf{e}A$	,	$C\mathsf{i}B$	:	$A\mathsf{o}C$	**Fresison**
	$B\mathsf{a}A$	,	$C\mathsf{e}B$	:	$A\mathsf{o}C$	**Camenop**

Reminder.

In syllogistics, all terms are nonempty.

Barbari. AaB, BaC: AiC.

Every unicorn is a white horse.
Every white horse is white.

Some unicorn is white.

In particular, this white unicorn exists.

The perfect moods.

Τέλειον μὲν οὖν καλῶ συλλογισμὸν τὸν μηδενὸς ἄλλου προσδεόμενον παρὰ τὰ εἰλημμένα πρὸς τὸ φανῆναι τὸ ἀναγκαῖον. *(An.Pr. I.i)*

Aristotle discusses the first figure in *Analytica Priora* I.iv, identifies **Barbara**, **Celarent**, **Darii** and **Ferio** as *perfect* and then concludes

Δῆλον δὲ καὶ ὅτι πάντες οἱ ἐν αὐτῷ συλλογισμοὶ τέλειοί εἰσι ... καλῶ δὲ τὸ τοιοῦτον σχῆμα πρῶτον. *(An.Pr. I.iv)*

Axioms of Syllogistics.

So the Axioms of Syllogistics according to Aristotle are:

Barbara. AaB, BaC : AaC
Celarent. AeB, BaC: AeC
Darii. AaB, BiC : AiC
Ferio. AeB, BiC: AoC

Simple and accidental conversion.

- Simple (*simpliciter*).
  - Xi$Y \rightsquigarrow Y$iX.
  - Xe$Y \rightsquigarrow Y$eX.
- Accidental (*per accidens*).
  - Xa$Y \rightsquigarrow X$iY.
  - Xe$Y \rightsquigarrow X$oY.

Syllogistic proofs (1).

We use the letters t_{ij} for terms and the letters k_i stand for copulae. We write a mood in the form

$$\frac{\begin{array}{c} t_{11}\ k_1\ t_{12} \\ t_{21}\ k_2\ t_{22} \end{array}}{t_{31}\ k_3\ t_{32}},$$

for example,

$$\frac{\begin{array}{c} \text{AaB} \\ \text{BaC} \end{array}}{\text{AaC}}$$

for **Barbara**. We write M_i for $t_{i1}\ k_i\ t_{i2}$ and define some operations on moods.

Syllogistic proofs (2).

- For $i \in \{1,2,3\}$, the operation s_i can only be applied if k_i is either 'i' or 'e'. In that case, s_i interchanges t_{i1} and t_{i2}.
- For $i \in \{1,2\}$, let p_i be the operation that changes k_i to its subaltern (if it has one), while p_3 is the operation that changes k_3 to its superaltern (if it has one).
- Let m be the operation that exchanges M_1 and M_2.
- For $i \in \{1,2\}$, let c_i be the operation that first changes k_i and k_3 to their contradictories and then exchanges M_i and M_3.
- Let per_π be the permutation π of the letters A, B, and C, applied to the mood.

$$\begin{array}{c} \text{BaA} \\ \text{AaC} \\ \hline \text{BaC} \end{array} \xrightarrow{\mathsf{per}} \begin{array}{c} \text{AaB} \\ \text{BaC} \\ \hline \text{AaC} \end{array}$$

Syllogistic proofs (3).

Given any set $\mathfrak{B}$ of "basic moods", a $\mathfrak{B}$-**proof** of a mood $M = M_1, M_2{:}M_3$ is a sequence $\langle \mathsf{o}_1, ..., \mathsf{o}_n \rangle$ of operations such that

- Only o_1 can be of the form c_1 or c_2 (but doesn't have to be).
- The sequence of operations, if applied to M, yields an element of $\mathfrak{B}$.

Syllogistic proofs (4).

$\langle \mathsf{s}_1, \mathsf{m}, \mathsf{s}_3, \mathsf{per}_{AC} \rangle$ is a proof of **Disamis** (from **Darii**) :

$$\frac{\begin{array}{c}\mathsf{AiB}\\ \mathsf{CaB}\end{array}}{\mathsf{AiC}} \xrightarrow{\mathsf{s}_1} \frac{\begin{array}{c}\mathsf{BiA}\\ \mathsf{CaB}\end{array}}{\mathsf{AiC}} \xrightarrow{\mathsf{m}} \frac{\begin{array}{c}\mathsf{CaB}\\ \mathsf{BiA}\end{array}}{\mathsf{AiC}} \xrightarrow{\mathsf{s}_3} \frac{\begin{array}{c}\mathsf{CaB}\\ \mathsf{BiA}\end{array}}{\mathsf{CiA}} \xrightarrow{\mathsf{per}} \frac{\begin{array}{c}\mathsf{AaB}\\ \mathsf{BiC}\end{array}}{\mathsf{AiC}}$$

$\langle \mathsf{s}_2 \rangle$ is a proof of **Datisi** (from **Darii**) :

$$\frac{\begin{array}{c}\mathsf{AaB}\\ \mathsf{CiB}\end{array}}{\mathsf{AiC}} \xrightarrow{\mathsf{s}_2} \frac{\begin{array}{c}\mathsf{AaB}\\ \mathsf{BiC}\end{array}}{\mathsf{AiC}}$$

Syllogistic proofs (5).

$\langle \mathsf{c}_1, \mathsf{per}_{\mathrm{BC}} \rangle$ is a proof of **Bocardo** by contradiction (from **Barbara**) :

$$\frac{\begin{array}{c}\mathsf{AoB}\\ \mathsf{CaB}\end{array}}{\mathsf{AoC}} \quad \xrightarrow{\mathsf{c}_1} \quad \frac{\begin{array}{c}\mathsf{AaC}\\ \mathsf{CaB}\end{array}}{\mathsf{AaB}} \quad \xrightarrow{\mathsf{per}} \quad \frac{\begin{array}{c}\mathsf{AaB}\\ \mathsf{BaC}\end{array}}{\mathsf{AaC}}$$

Syllogistic proofs (6).

Let $\mathfrak{B}$ be a set of moods and M be a mood. We write $\mathfrak{B} \vdash M$ if there is $\mathfrak{B}$-proof of M.

Mnemonics (1).

Bárbara, Célarént, Darií, Ferióque prióris,
Césare, Cámestrés, Festíno, Baróco secúndae.
Tértia Dáraptí, Disámis, Datísi, Felápton,
Bocárdo, Feríson habét. Quárta ínsuper áddit
Brámantíp, Camenés, Dimáris, Fesápo, Fresíson.

“These words are more full of meaning than any that were ever made.” (Augustus de Morgan)

Mnemonics (2).

- The fi rst letter indicates to which one of the four perfect moods the mood is to be reduced: ‘B’ to Barbara, ‘C’ to Celarent, ‘D’ to Darii, and ‘F’ to Ferio.
- The letter ‘s’ after the ith vowel indicates that the corresponding proposition has to be simply converted, *i.e.*, a use of s_i.
- The letter ‘p’ after the ith vowel indicates that the corresponding proposition has to be accidentally converted (“*per accidens*”), *i.e.*, a use of p_i.
- The letter ‘c’ after the fi rst or second vowel indicates that the mood has to be proved indirectly by proving the contradictory of the corresponding premiss, *i.e.*, a use of c_i.
- The letter ‘m’ indicates that the premises have to be interchanged (“moved”), *i.e.*, a use of m.
- All other letters have only aesthetic purposes.

A metatheorem.

We call a proposition negative if it has either ‘e’ or ‘o’ as copula.

Theorem (Aristotle). If M is a mood with two negative premises, then

$$\mathfrak{B}_{\mathrm{BCDF}} \not\models M.$$

Metaproof (1).

Suppose $\mathsf{o} := \langle \mathsf{o}_1, ..., \mathsf{o}_n \rangle$ is a $\mathfrak{B}_{\mathrm{BCDF}}$-proof of M.

- The s-rules don't change the copula, so if M has two negative premises, then so does $\mathsf{s}_i(M)$.
- The superaltern of a negative proposition is negative and the superaltern of a positive proposition is positive. Therefore, if M has two negative premises, then so does $\mathsf{p}_i(M)$.
- The m-rule and the per-rules don't change the copula either, so if M has two negative premises, then so do $\mathsf{m}(M)$ and $\mathsf{per}_\pi(M)$.

As a consequence, if $\mathsf{o}_1 \neq \mathsf{c}_i$, then $\mathsf{o}(M)$ has two negative premisses.We check that none of **Barbara**, **Celarent**, **Darii** and **Ferio** has two negative premisses, and are done, as o cannot be a proof of M.

Metaproof (2).

So, $\mathsf{o}_1 = \mathsf{c}_i$ for either $i = 1$ or $i = 2$. By definition of c_i, this means that the contradictory of one of the premisses is the conclusion of $\mathsf{o}_1(M)$. Since the premisses were negative, the conclusion of $\mathsf{o}_1(M)$ is positive. Since the other premiss of M is untouched by o_1, we have that $\mathsf{o}_1(M)$ has at least one negative premiss and a positive conclusion. The rest of the proof $\langle \mathsf{o}_2, ..., \mathsf{o}_n \rangle$ may not contain any instances of c_i.

Note that none of the rules s, p, m and per change the copula of the conclusion from positive to negative.

So, $\mathsf{o}(M)$ still has at least one negative premiss and a positive conclusion. Checking **Barbara**, **Celarent**, **Darii** and **Ferio** again, we notice that none of them is of that form. Therefore, o is not a $\mathfrak{B}_{\mathrm{BCDF}}$-proof of M. Contradiction. q.e.d.

Other metatheoretical results.

- If M has two particular premises (i.e., with copulae 'i' or 'o'), then $\mathrm{BCDF} \not\models M$ (**Exercise 15**).
- If M has a positive conclusion and one negative premiss, then $\mathrm{BCDF} \not\models M$.
- If M has a universal conclusion (i.e., with copula 'a' or 'e') and one particular premiss, then $\mathrm{BCDF} \not\models M$.

Aristotelian modal logic.

Modalities.

- $\mathbf{A}p \doteq$ "p" (no modality, "assertoric").
- $\mathbf{N}p \doteq$ "necessarily p".
- $\mathbf{P}p \doteq$ "possibly p" (equivalently, "not necessarily not p").
- $\mathbf{C}p \doteq$ "contingently p" (equivalently, "not necessarily not p and not necessarily p").

Every (assertoric) mood $p, q : r$ represents a modal mood $\mathbf{A}p, \mathbf{A}q : \mathbf{A}r$. For each mood, we combinatorially have $4^3 = 64$ modalizations, i.e., $256 \times 64 = 16384$ modal moods.

Modal conversions.

- **Simple.**
  - $\mathbf{N}X\mathrm{e}Y \rightsquigarrow \mathbf{N}Y\mathrm{e}X$
  - $\mathbf{N}X\mathrm{i}Y \rightsquigarrow \mathbf{N}Y\mathrm{i}X$
  - $\mathbf{C}X\mathrm{e}Y \rightsquigarrow \mathbf{C}Y\mathrm{e}X$
  - $\mathbf{C}X\mathrm{i}Y \rightsquigarrow \mathbf{C}Y\mathrm{i}X$
  - $\mathbf{P}X\mathrm{e}Y \rightsquigarrow \mathbf{P}Y\mathrm{e}X$
  - $\mathbf{P}X\mathrm{i}Y \rightsquigarrow \mathbf{P}Y\mathrm{i}X$
- **Accidental.**
  - $\mathbf{N}X\mathrm{a}Y \rightsquigarrow \mathbf{N}X\mathrm{i}Y$
  - $\mathbf{C}X\mathrm{a}Y \rightsquigarrow \mathbf{C}X\mathrm{i}Y$
  - $\mathbf{P}X\mathrm{a}Y \rightsquigarrow \mathbf{P}X\mathrm{i}Y$
  - $\mathbf{N}X\mathrm{e}Y \rightsquigarrow \mathbf{N}X\mathrm{o}Y$
  - $\mathbf{C}X\mathrm{e}Y \rightsquigarrow \mathbf{C}X\mathrm{o}Y$
  - $\mathbf{P}X\mathrm{e}Y \rightsquigarrow \mathbf{P}X\mathrm{o}Y$
- **Relating to the symmetric nature of contingency.**
  - $\mathbf{C}X\mathrm{i}Y \rightsquigarrow \mathbf{C}X\mathrm{e}Y$
  - $\mathbf{C}X\mathrm{e}Y \rightsquigarrow \mathbf{C}X\mathrm{i}Y$
  - $\mathbf{C}X\mathrm{a}Y \rightsquigarrow \mathbf{C}X\mathrm{o}Y$
  - $\mathbf{C}X\mathrm{o}Y \rightsquigarrow \mathbf{C}X\mathrm{a}Y$
- $\mathbf{N}X\mathrm{x}Y \rightsquigarrow \mathbf{A}X\mathrm{x}Y$
  (Axiom $\mathbf{T}$: $\Box\varphi \rightarrow \varphi$)

Modal axioms.

What are the “perfect modal syllogisms”?

- Valid assertoric syllogisms remain valid if $\mathbf{N}$ is added to all three propositions.

  Barbara $(A\mathrm{a}B, B\mathrm{a}C{:}A\mathrm{a}C) \rightsquigarrow$ **NNN Barbara** $(\mathbf{N}A\mathrm{a}B, \mathbf{N}B\mathrm{a}C{:}\mathbf{N}A\mathrm{a}C)$.

  First complications in the arguments for **Bocardo** and **Baroco**.

- By our conversion rules, the following can be added to valid assertoric syllogisms:
  - $\mathbf{NNA}$,
  - $\mathbf{NAA}$,
  - $\mathbf{ANA}$.
- Anything else is problematic.

The “two Barbaras”.

NAN Barbara

$$\frac{\mathbf{N}A\mathrm{a}B \quad \mathbf{A}B\mathrm{a}C}{\mathbf{N}A\mathrm{a}C}$$

ANN Barbara

$$\frac{\mathbf{A}A\mathrm{a}B \quad \mathbf{N}B\mathrm{a}C}{\mathbf{N}A\mathrm{a}C}$$

From the modern point of view, both modal syllogisms are invalid, yet Aristotle claims that **NAN Barbara** is valid, but **ANN Barbara** is not.

De dicto versus *De re*.

We interpreted $\mathbf{N}A\mathrm{a}B$ as

"The statement '$A\mathrm{a}B$' is necessarily true.'

(*De dicto* interpretation of necessity.)

Alternatively, we could interpret $\mathbf{N}A\mathrm{a}B$ *de re* (Becker 1933):

"Every B happens to be something which is necessarily an A."

Aristotelian temporal logic: the sea battle

According to the square of oppositions, exactly one of “it is the case that p” and “it is not the case that p” is true.

Either “it is the case that there will be a sea battle tomorrow”

or “it is not the case that there will be a sea battle tomorrow”.

Problematic for existence of free will, and for Aristotelian metaphysics.

The Master argument.

Diodorus Cronus (IVth century BC).

- Assume that p is not the case.
- In the past, “It will be the case that p is not the case” was true.
- In the past, “It will be the case that p is not the case” was necessarily true.
- Therefore, in the past, “It will be the case that p” was impossible.
- Therefore, p is not possible.

Ergo: Everything that is possible is true.