Scholasticism.

The XIIIth century: the Golden Age of Scholasticism.

- Reasoning and analysis (involving logic, metaphysics and semantics), based on authorities: philological and logical analysis of original texts.
- Forms: *quaestiones*, *disputationes*.

Logica nova.

- *insolubilia*: fallacies and paradoxes.
- *syncategoremata*: and, or, not, if, every, some, only, except.
- *obligationes*: a game-theoretic approach to logic.
- “Terminist logic”: *proprietates terminorum*.

Logic in the XIIth/XIIIth century.

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- Petrus Hispanus, later Pope John XXI. (?) , (c.1205-1277), *Summulae Logicales*.
- Robert Kilwardby (c.1215-1279). Modal syllogistic conversion rules.
- Roger Bacon (1214-1292).

Logic in the XIIIth/XIVth century.

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- Johannes Duns Scotus (1266-1308). *Doctor Subtilis*.
- *The pseudo-Scot*. New modalities: *dubium*, *scitum*, *opinatum*, *volitum*, *dilectum*.
- William Ockham (c.1295-1349). *Entia non sunt multiplicanda praeter necessitatem.*

Via antiqua / via moderna.

XIVth and XVth century. Philosophy sharply divided into *via antiqua* and *via moderna*.

Via Antiqua.

- *logica vetus*
- Thomistic realism.

Via Moderna.

- *logica nova*.
- Semantical analysis.
- Nominalism.

- The Terminists.
- The Modists (XIIIth / XIVth century).
  - “speculative grammar” based on *modi*.
  - Boëthius of Dacia (d.1290)
  - Pierre d’Auvergne (d.1303)
  - Martin of Dacia (d.1304)
  - Thomas of Erfurt (c.1330)
  - Johannes Aurifaber (c.1330)

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- Nominalism.

- The Terminists.
- The Modists (XIIIth / XIVth century).
- Walter Burley (c.1275-1344).
- William Ockham (c.1295-1349).

Terministic logic (1).

Moving from analysis of meaning in words (what does *homo* mean?) to analysis of meaning of terms in phrases (what part of the meaning of *homo* is responsible for the fact that "*omnis homo mortalis est*" is true?).

- Syllogistics doesn't analyse the truth-status of categorial propositions any further.
- Linguistic analysis (predication vs non-predication) at the basis of the theory of categories.
- Grammar investigated the meaning of single words (outside of the context of propositions).
- Origins in the school of Chartres (c.1030): 'contextual approach' (de Rijk, 1967).

Terministic logic (2).

Subtle questions.

- Compare *“homo est animal”*, *“homo est species”*, and *“homo est disyllabum”*.
  In each of the cases, the meaning of *homo* is slightly different.
- What do qualifiers do with meanings?
  If I go from *“omnis homo est philosophus”* to *“paene omnis homo est philosophus”*, how does the explanation for the meaning change?

Syncategoremata.

- Grammarians' definition. A term is a categorema if it can be the subject or the predicate of a proposition. Other meaningful terms are syncategoremata.
- **Example 1.** *Socrates currit*.
- **Example 2.** *Socrates non currit*.
- Logicians' definition. An incomplete list of about fifty words that are discussed as syncategorematic.Among them are words like omnis.
- Important syncategoremata: *et*, *ut*, *cum*, *vel*, *omnis*, *uterque*...

Suppositio (1).

- An analysis of the meaning of terms in propositions: *Suppositio* as a theory of reference.
- **Situation 1.**
  - Under what conditions is *omnis homo philosophus est* true?
  - If *philosophus* supposits for every instance of *homo* (*suppositio mobilis*).
  - *Instantiation: Aristoteles homo est. Aristoteles philosophus est.*
- **Situation 2.**
  - Under what conditions is *omnis homo praeter Socratem philosophus est* true?
  - If *philosophus* supposits for every instance of *homo* except for Socrates.
  - *Instantiation: Aristoteles homo est.* ~~*Aristoteles praeter Socrates philosophus est.*~~ (*suppositio immobilis*).

An aside.

- Latin doesn’t have an indefi nite article.
  - *Homo est philosophus.*
  - A man is a philosopher.
  - (Some man is a philosopher.)
  - *Aliquis homo est philosophus.*
- The medievals didn’t use quotation marks.
  - *Homo est disyllabum.*
  - ‘Human’ is bisyllabic.

Suppositio (2).

- **Situation 3.**
  - Under what conditions is *homo est disyllabum* true?
  - If *disyllabum* supposits for every instance of *homo*. (But here, *homo* is a singular term standing for '*homo*').
  - *Flawed instantiation: Aristoteles homo est.* ~~*Aristoteles disyllabum est.*~~ (*suppositio materialis*).
- Consequences for logic: Whether conversion rules can be applied depends on the type of supposition in the proposition.

homo est disyllabum.

aliquis homo est disyllabum.

aliquis disyllabum est homo. (simple conversion)

disyllabum est homo.

Bisyllabic is a man.

Suppositio (3).

Types of *suppositio* (Spade 1982):

- *suppositio impropria*.
- *suppositio propria*.
  - *suppositio materialis*.
  - *suppositio formalis*.
    - *suppositio discreta*.
    - *suppositio simplex*.
    - *suppositio personalis*.
      - *suppositio determinata*.
      - *suppositio confusa tantum*.
      - *suppositio mobilis*.
      - *suppositio immobilis*.

Paul Vincent **Spade**, Thoughts, Words and Things: An Introduction to Late Mediaeval Logic and Semantic Theory, *preprint*

`http://www.pvspade.com/Logic/`

Suppositio (4).

- What makes *Aristoteles academicus erat* true?
- *Attempt 1.* If *academicus* supposits for *Aristoteles*. But if *academicus* supposits for *Aristoteles*, then *Aristoteles academicus est* is true.
- *Attempt 2* (modern reading). If there was a point in the past when *academicus* supposited for *Aristoteles*.
- Medieval theory: ampliation and restriction: *si terminus communis verbo de praeterito supponeret, posset supponere pro non-enti, ut hoc* homo cucurrit *verum est pro Caesare* (William of Shyreswood, *Introductiones*).
- In general: the predicate determines the type of *suppositio* and whether *ampliatio* has to be used in order to determine the truth conditions.

Fallacies: *secundum quid et simpliciter.*

Around 1120, Boëthius' translation of the *Sophistici Elenchi* is rediscovered. Aristotelian discussions of fallacies.

The Oathbreaker:

- **Oath.** I shall never leave Rome. I shall become an oathbreaker.
- **Fact.** I have left Rome.

Argument. Since I have left Rome, I broke my oath. Since I have broken my oath, I have kept my oath. ~~I am an oathbreaker and an oathkeeper at the same time.~~ I am an oathbreaker and an oathkeeper.

secundum quid et simpliciter

- *simpliciter.* An oathbreaker is a person who breaks at least one oath.
- *secundum quid.* An oathkeeper is a person who keeps the oath.

Insolubles (1).

The most famous insoluble: **the Liar**.

This sentence is false.

φ : φ is false.

In the early literature on insolubles, there are five solutions to this paradox:

- *secundum quid et simpliciter*.
- *transcasus*.
- Distinction between the exercised act and the signified act.
- *restrictio*.
- *cassatio*.

Insolubles (2).

- *secundum quid et simpliciter*.
  - Mentioned by Aristotle (*Sophistici Elenchi*, 180b2-3).

Insolubles (2).

- *secundum quid et simpliciter*.
  Solution. Unclear.
- *transcasus*.
  - Derives from the Stoic *metaptosis*: differing truth-values over time.
  - When I say “I am speaking a falsehood” I am referring to what I said immediately preceding to that sentence.
  - If I didn’t say anything before that, then the sentence is just false.

Insolubles (2).

- *secundum quid et simpliciter*.
  Solution. Unclear.
- *transcasus*.
  Solution. The Liar sentence is false.
- Distinction between the exercised act and the signified act.
  - Johannes Duns Scotus, *Questiones.*
  - The exercised act of the liar is “speaking the truth”.
  - The signifi ed act of the liar is “speaking a falsehood”.
  - The liar expresses something which is not the truth, so it is false.

Insolubles (2).

- *secundum quid et simpliciter*.
  Solution. Unclear.
- *transcasus*.
  Solution. The Liar sentence is false.
- Distinction between the exercised act and the signified act.
  Solution. The Liar sentence is false.
- *restrictio*.
  - The *restringentes* do not allow assignment of truth-values to sentences with self-reference.
  - Not only the Liar, but also the following insoluble: φ : ψ is false. ψ : φ is false (linked liars)

Insolubles (2).

- *secundum quid et simpliciter*.
  Solution. Unclear.
- *transcasus*.
  Solution. The Liar sentence is false.
- Distinction between the exercised act and the signified act.
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- *restrictio*.
  - The *restringentes* do not allow assignment of truth-values to sentences with self-reference.
  - Not only the Liar, but also the following insoluble: φ : ψ is false. ψ : φ is false ... and ... “This sentence has fi ve words.”

Insolubles (2).

- *secundum quid et simpliciter*.
  Solution. Unclear.
- *transcasus*.
  Solution. The Liar sentence is false.
- Distinction between the exercised act and the signified act.
  Solution. The Liar sentence is false.
- *restrictio*.
  Solution. The Liar sentence does not have a truth value.
- *cassatio*.
  - If you are uttering an insoluble, you are saying nothing.
  - Therefore an insoluble has the same truth value as the empty utterance: none.

Insolubles (2).

- *secundum quid et simpliciter*.
  Solution. Unclear.
- *transcasus*.
  Solution. The Liar sentence is false.
- Distinction between the exercised act and the signified act.
  Solution. The Liar sentence is false.
- *restrictio*.
  Solution. The Liar sentence does not have a truth value.
- *cassatio*.
  Solution. The Liar sentence does not have a truth value.

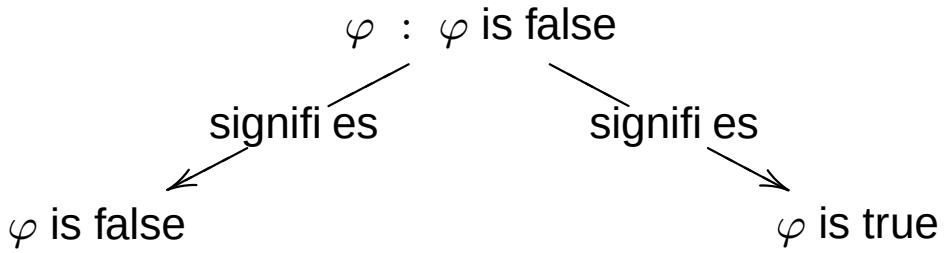
Insolubles (3).

- The most productive era in the theory of insolubles was from 1320 to 1350.
- Thomas Bradwardine (c.1295-1349).
- Roger Swyneshed (mid XIVth century).
- William Heytesbury (c.1310-1372).
- John Wyclif (c.1330-1384).
- Peter of Ailly (*Petrus de Alliaco*; 1350-1420).

Bradwardine.

Thomas Bradwardine (c.1295-1349).

- *Insolubilia*: 1321-1324.
- *Adverbial Theory of propositional signification* (Spade).
- Every sentence signifies that it is true.
- A sentence is true if and only if everything that it signifies is true (*sicut est*). A sentence is false if and only if there is something that it signifies which is false (*aliter quam est*).
- The Liar sentence signifies that it is false.



Swyneshed.

Roger Swyneshed (mid XIVth century).

- A sentence is true if and only if it signifies *sicut est* and if it not self-falsifying. Self-falsifying sentences are always false.
- The Liar is self-falsifying, so it is false.
- *Consequence of Swyneshed's definition of truth.*
  - φ : φ is false.
  - ψ : φ is not false.
  - φ is false as it is self-falsifying. But then ψ is false, too. But φ and ψ are contradictories.

Heytesbury.

William Heytesbury (c.1310-1372).

- *1335. Regulae solvendi sophismata.*
- *The source of the paradox according to Heytesbury:* The Liar "φ : φ is false" is only paradoxical since we want to retain the usual theory of signification for it. If we give that up, there is no paradox. For example, φ could signify "*Socrates currit*" which is free of paradoxes.
- But φ cannot be evaluated according to the usual theory of signification. Therefore, anyone who utters φ must have some other hidden signification in mind. There is no way to analyze φ further before we know which one this is.

Sophismata and semantics.

Some of the problems concerning the semantics of syncategoremata are part of the theory of sophismata:

Socrates bis videt $\left($*omnem hominem praeter Platonem*$\right)$.

- **Scenario 1.** Socrates enters the room and sees everyone. He leaves. Plato leaves the room. Socrates returns and sees everyone except for Plato.
  Socrates videt Platonem.
- **Scenario 2.** Plato is not in the room at all. Socrates enters the room twice and sees everyone in there.
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Dialogic Logic (1).

Paul Lorenzen (1958): Explaining the meaning of propositional connectives via games and strategies.

- Two players, the **Proponent** and the **Opponent**.
- In the round 0, the Proponent has to assert the formula to be proved and the Opponent can make as many assertions as he wants. After that, the opponent starts the game.
- In all other moves, the players have to do an *announcement* and an *action*.
- An **announcement** is either of the form $\mathbf{attack}(n)$ or of the form $\mathbf{defend}(n)$, interpreted as "I shall attack the assertion made in round n" and "I shall defend myself against the attack made in round n".

Dialogic Logic (2).

- An **action** can be one of the following moves:
  - $\mathbf{assert}(\Phi)$,
  - $\mathbf{which\ one?}$,
  - $\mathbf{left?}$,
  - $\mathbf{right?}$,
  - $\mathbf{what\ if?}, \mathbf{assert}(\Phi)$.
- You can only attack lines in which the other player asserted a formula. Depending on the formula, the following attacks are allowed:
  - $\Phi \vee \Psi$ may be attacked by $\mathbf{which\ one?}$,
  - $\Phi \wedge \Psi$ may be attacked by $\mathbf{left?}$ or $\mathbf{right?}$,
  - both $\Phi \rightarrow \Psi$ and $\neg\Phi$ may be attacked by "$\mathbf{what\ if?}, \mathbf{assert}(\Phi)$".

Dialogic Logic (3).

- You can only defend against a line in which the other player attacked. Depending on the attack, the following defenses are allowed:
  - If $\Phi \vee \Psi$ was attacked by $\mathbf{which\ one?}$, you may defend with either $\mathbf{assert}(\Phi)$ or $\mathbf{assert}(\Psi)$.
  - If $\Phi \wedge \Psi$ was attacked by $\mathbf{left?}$, you may defend with $\mathbf{assert}(\Phi)$, if it was attacked by $\mathbf{right?}$, you may defend with $\mathbf{assert}(\Psi)$.
  - If $\Phi \rightarrow \Psi$ was attacked by "$\mathbf{what\ if?}, \mathbf{assert}(\Phi)$", you may defend with $\mathbf{assert}(\Psi)$.
  - You cannot defend an attack on $\neg\Phi$.

Dialogic Logic (4).

The rules of the (constructive) game:

- In each move, the action and the announcement have to fi t together, i.e., if the player announces $\mathbf{attack}(n)$ or $\mathbf{defend}(n)$, then the action has to be an attack on move n or a defense against move n.
- In round $n+1$, the Opponent has to either attack or defend against round n.
- An attack is called open if it has not yet been defended.
- The Proponent may attack any round, but may only defend against the most recent open attack. He may use any defense or attack against a given round at most once.
- The Opponent may assert any atomic formulas.
- The Proponent may assert only atomic formulas that have been asserted by the Opponent before.

Dialogic logic (5).

If one player cannot make any legal moves anymore, the other player has won.

Example 1.

0		—		$\mathbf{assert}(p \wedge q \rightarrow q \wedge p)$
1	$\mathbf{attack}(0)$	$\mathbf{what\ if?}\ \mathbf{assert}(p \wedge q)$		
2			$\mathbf{attack}(1)$	$\mathbf{left?}$
3	$\mathbf{defend}(2)$	$\mathbf{assert}(p)$		
4			$\mathbf{attack}(1)$	$\mathbf{right?}$
5	$\mathbf{defend}(4)$	$\mathbf{assert}(q)$		
6			$\mathbf{defend}(1)$	$\mathbf{assert}(q \wedge p)$
7	$\mathbf{attack}(6)$	$\mathbf{left?}$		
8			$\mathbf{defend}(7)$	$\mathbf{assert}(q)$
9	—	—		

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2			$\mathbf{attack}(1)$	$\mathbf{left?}$
3	$\mathbf{defend}(2)$	$\mathbf{assert}(p)$		
4			$\mathbf{attack}(1)$	$\mathbf{right?}$
5	$\mathbf{defend}(4)$	$\mathbf{assert}(q)$		
6			$\mathbf{defend}(1)$	$\mathbf{assert}(q \wedge p)$
7	$\mathbf{attack}(6)$	$\mathbf{right?}$		
8			$\mathbf{defend}(7)$	$\mathbf{assert}(p)$
9	—	—		

Dialogic logic (6).

We say that Φ is **dialogically valid** if the Proponent has a winning strategy in the game in which he asserts Φ in round 0.
In symbols: $\models_{\text{dialog}} \Phi$.
The dialogically valid formulas are exactly those provable in intuitionistic propositional logic.

Example 2.

0		—		$\mathbf{assert}(\neg\neg p \to p)$
1	$\mathbf{attack}(0)$	$\mathbf{what\ if?\ assert}(\neg\neg p)$		
2			$\mathbf{attack}(1)$	$\mathbf{what\ if?\ assert}(\neg p)$
3	$\mathbf{attack}(2)$	$\mathbf{what\ if?\ assert}(p)$		
4			—	—

Dialogic Logic (7).

The rules of the (classical) game:

- In each move, the action and the announcement have to fit together, i.e., if the player announces $\mathbf{attack}(n)$ or $\mathbf{defend}(n)$, then the action has to be an attack on move n or a defense against move n.
- In round $n+1$, the Opponent has to either attack or defend against round n.
- The Proponent may attack and defend against any round. He may use any defense or attack against a round at most once.
- The Opponent may assert any atomic formulas.
- The Proponent may assert only atomic formulas that have been asserted by the Opponent before.

Dialogic logic (8).

We say that Φ is **classically valid** if the Proponent has a winning strategy in the (classical) game in which he asserts Φ in round 0.
In symbols: $\models_{\text{class}} \Phi$.
The classically valid formulas are exactly those provable in classical propositional logic.

Example 2a.

0		—		$\mathbf{assert}(\neg\neg p \rightarrow p)$
1	$\mathbf{attack}(0)$	$\mathbf{what\ if?\ assert}(\neg\neg p)$		
2			$\mathbf{attack}(1)$	$\mathbf{what\ if?\ assert}(\neg p)$
3	$\mathbf{attack}(2)$	$\mathbf{what\ if?\ assert}(p)$		
4			$\mathbf{defend}(1)$	$\mathbf{assert}(p)$
5	—	—		