DEFINABLE CLUBS

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ABSTRACT. In my talk, I want to present results from an ongoing project with
Omer Ben-Neria (Jerusalem) that studies definable closed unbounded subsets
of uncountable cardinals, focusing on large cardinals and singular cardinals.
Our results show that, for certain collections of formulas and classes of param-
eters, important combinatorial properties of the given cardinal are reflected in
the structural features of the collection of all closed unbounded sets definable
in the given way. For example, the assumption that the first limit cardinal $\aleph_\omega$
is Jónsson can be shown to have various non-trivial implications on the col-
lection of closed unbounded subsets of $\aleph_\omega$ that can be defined by Σ_1-formulas
with parameters in $H(\aleph_\omega) \cup \{\aleph_\omega\}$, and these consequences can then be used
to restrict the class of possible models of set theory in which $\aleph_\omega$ is Jónsson.