Mathematical Statistics I

The prerequisite for these lectures is some basic knowledge in measure theoretic probability. It is assumed that notions like *σ-algebra, measure, Lebesgue measure, measure integral, independence, weak law of large numbers, central limit theorem, Markov kernel, Fubini's theorem etc.* are known.
The lectures offer basic results on estimation and testing for parametric statistical models. They are accompanied by exercises (Übungen) which are an indispensable part of the studies.

Content

1. Optimal unbiased estimators
2. The substitution principle for estimators and Maximum-Likelihood estimators
3. The multivariate normal distribution
4. The general linear model. (Basic results on testing and estimation without optimality considerations, e.g. the Gauss-Markov theorem for estimators and Fishers F-test)
5. Optimal test for simple hypotheses (Neyman-Pearson Lemma)
6. Monotone likelihood ratio and optimal one-sided tests
7. Optimal two-sided tests
8. Conditional expectation and conditional experiments (Radon-Nikodym theorem)
9. Sufficiency and completeness and their application for estimating and testing
10. Exponential families
11. Testing in $k+1$-parametric exponential families: Conditional tests
12. Transformation to unconditional tests
13. Testing for the mean and the variance in normal families (Optimality of Student's t-test etc.)
14. Duality of confidence regions and families of tests.

Literature

1. Bickel, P. and Doksum,K.(1977) Mathematical Statistics. Holden-Day, Düsseldorf
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5. Pfanzagl, J.(1994) Parametric Statistical Theory. De Gruyter, Berlin
6. Witting, H.(1974) Mathematische Statistik.Teubner, Stuttgart
7. Witting, H.(1985) Mathematische Statistik I. Teubner Stuttgart