Bsp Konvex

$f(x) = x^2$ ist konvex

Zeige: $f(\alpha x + (1-\alpha) y)$

$\leq \alpha f(x) + (1-\alpha) f(y)$

$\forall x, y \in D_f = \mathbb{R}, \ \alpha \in [0,1]$

$$(\alpha x + (1-\alpha) y)^2 \overset{!}{\leq} \alpha x^2 + (1-\alpha) y^2$$

$$\parallel$$

$$\alpha^2 x^2 + 2\alpha(1-\alpha) xy + (1-\alpha)^2 y^2$$

$$= \alpha^2 x^2 + 2\alpha(1-\alpha) xy + (1-\alpha)^2 y^2$$
$$+ \alpha x^2 + (1-\alpha) y^2$$
$$- \alpha x^2 \mp (1-\alpha) y^2 \quad \leq 0$$

$$\leq \alpha x^2 + (1-\alpha) y^2$$

Zeige noch: ≤ 0

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$$G = \alpha(\alpha-1)\left[x^2 - 2xy + y^2\right]$$

$$= \underbrace{\alpha}_{\geq 0}\underbrace{(\alpha-1)}_{\leq 0}\underbrace{(x-y)^2}_{\geq 0} \leq 0$$

Bsp Periodische Funktion

$$f(x) = \sin(\omega x) \qquad \omega > 0$$

$$f(x+\alpha) = \sin(\omega(x+\alpha)) \overset{!}{=} f(x) = \sin(\omega x)$$

$$\omega(x+\alpha) = \omega x + 2k\pi$$

$$\rightarrow \qquad \alpha = \frac{2k\pi}{\omega}$$

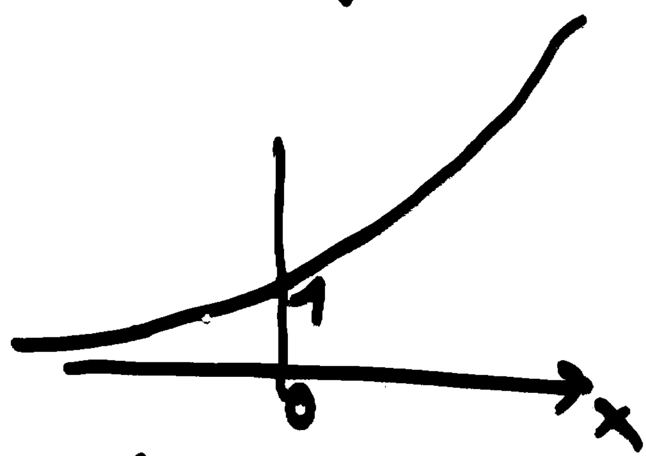
Primitive Periode $\alpha^* = \dfrac{2\pi}{\omega}$

Grundfunktionen

1.) $f(x) = e^x$

$D_f = \mathbb{R}$

e Euler Zahl

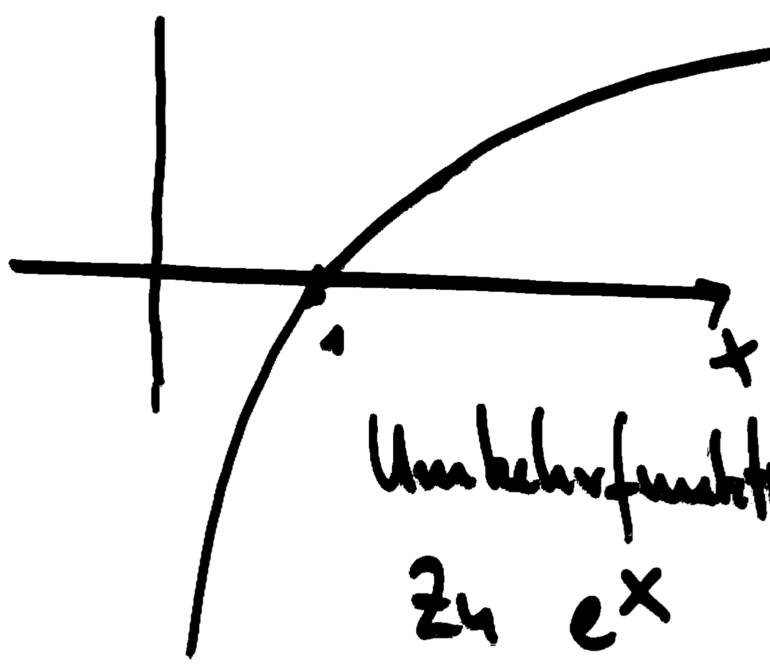


Exponentialfunktion

2.) $f(x) = \ln x$

$D_f = \mathbb{R}^+ \equiv \mathbb{R}_{>0}$

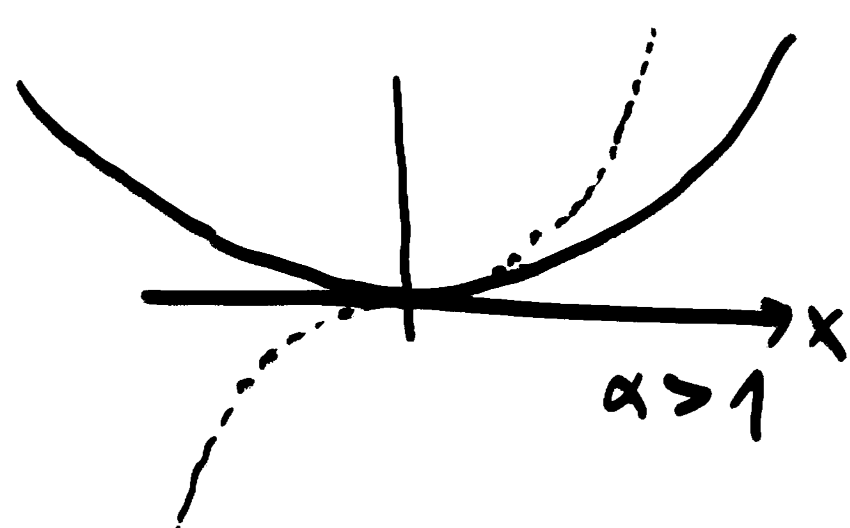
Natürlicher Logarithmus



Umkehrfunktion zu e^x

3.) $f(x) = x^\alpha$

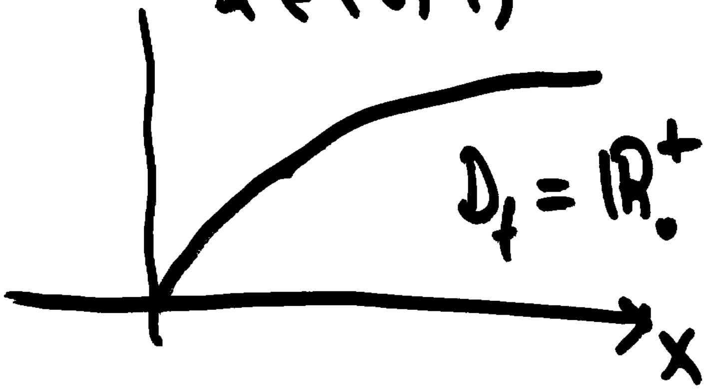
Potenzfunktion

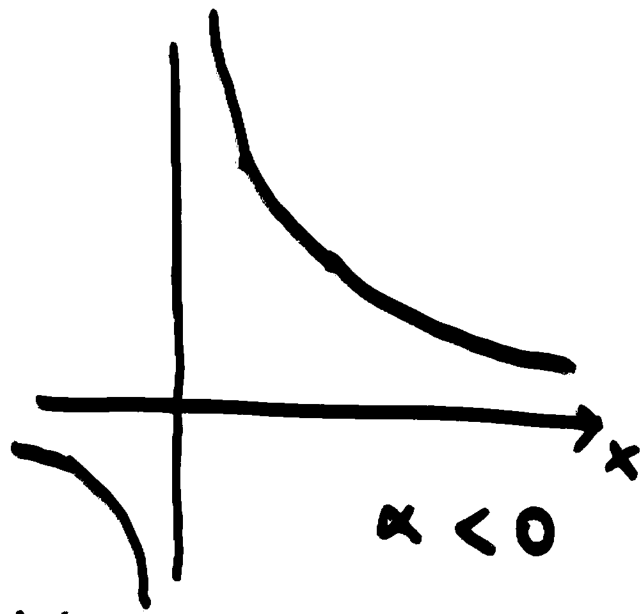


3.) weiter

$\alpha \in (0,1)$

$D_f = \mathbb{R}_0^+$





$$D_f = \begin{cases} \mathbb{R}, & \alpha \in \mathbb{N} \\ \mathbb{R}\setminus\{0\}, & \alpha \in \mathbb{Z},\ \alpha < 0 \\ \mathbb{R}^+, & \text{sonst.} \end{cases}$$

$x^\alpha = e^{\alpha \ln x}$ im Fall "sonst"

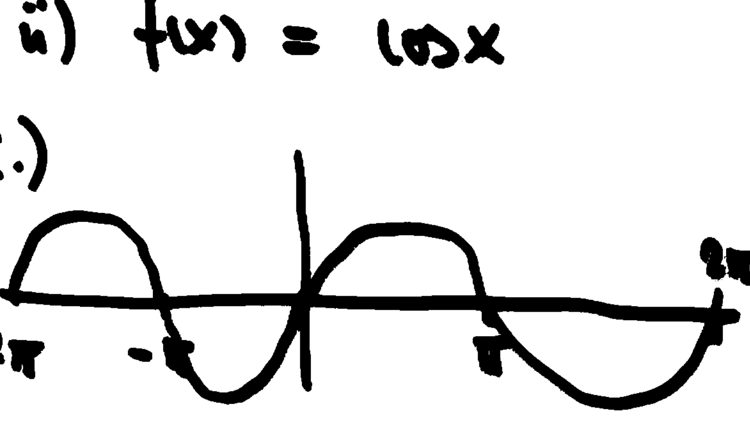
4.) $\underbrace{\text{Trigonometrische Funktionen}}_{\text{Grund}}$

i.) $f(x) = \sin x$

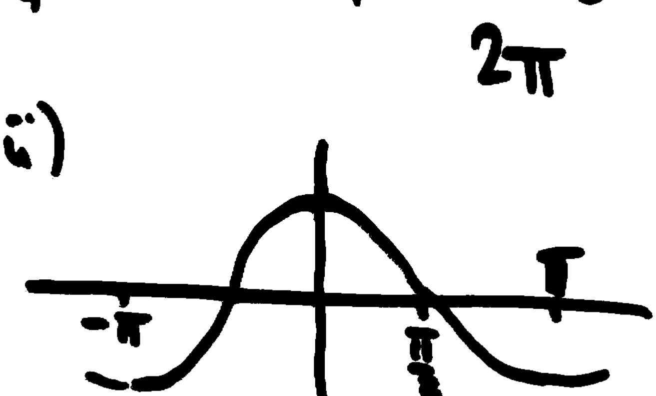
ii) $f(x) = \cos x$

$D_f = \mathbb{R}$, Periode 2π

i.)



ii)



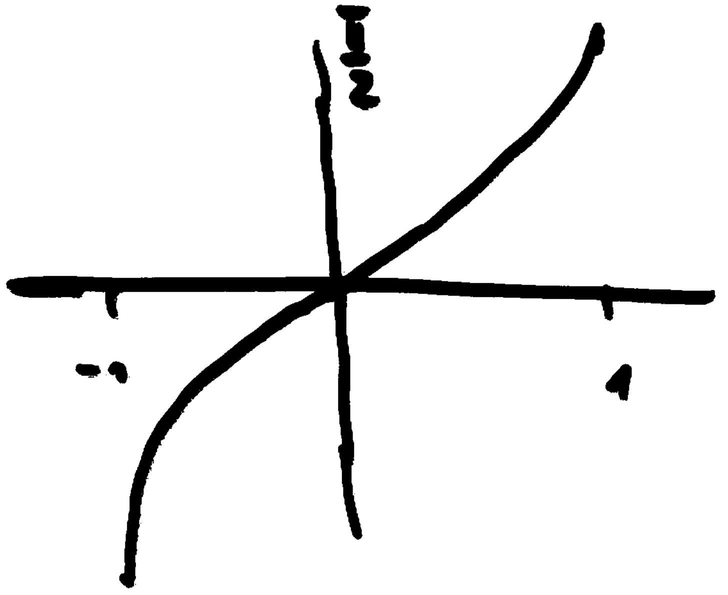
5.) Arcus Funktionen

(= Umkehrfunktionen von sin, cos)

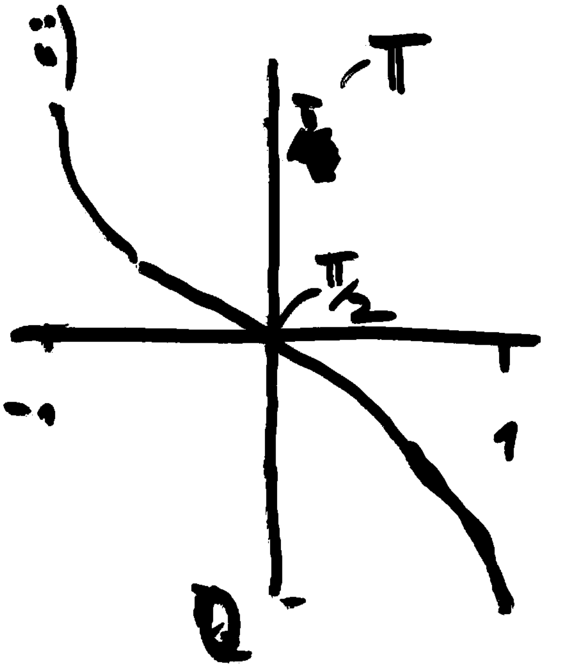
i.) $f(x) = \arcsin x$ $\quad D_f = [-1, 1]$

ii.) $f(x) = \arccos x$ $\quad D_f = [-1, 1]$

i.)



ii.)



<u>Merke</u>: Elementare Funktionen gehen aus Grundfunktionen durch Komposition, Multiplikation, Division, etc hervor.

Bsp: i.) $f(x) = a^x := e^{x \ln a}$ $\quad a > 0$

"Exponentialfunktion zur Basis a"

ii) Umkehrfunktion zu i.)

"Logarithmus zur Basis a"

$$\log_a x := \frac{1}{\ln a} \ln x$$

iii) $\operatorname{artanh}(x) = \frac{1}{2} \ln \frac{x+1}{-(x-1)} = \frac{1}{2} \ln \frac{1+x}{1-x}$ $\quad D = (-1,1)$

<u>Nachweis</u>: $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = y$

Auflösen nach x:

$y(e^x + e^{-x}) = e^x - e^{-x} \quad | \cdot e^x \quad \rightarrow$

$y(e^{2x} + 1) = e^{2x} - 1$

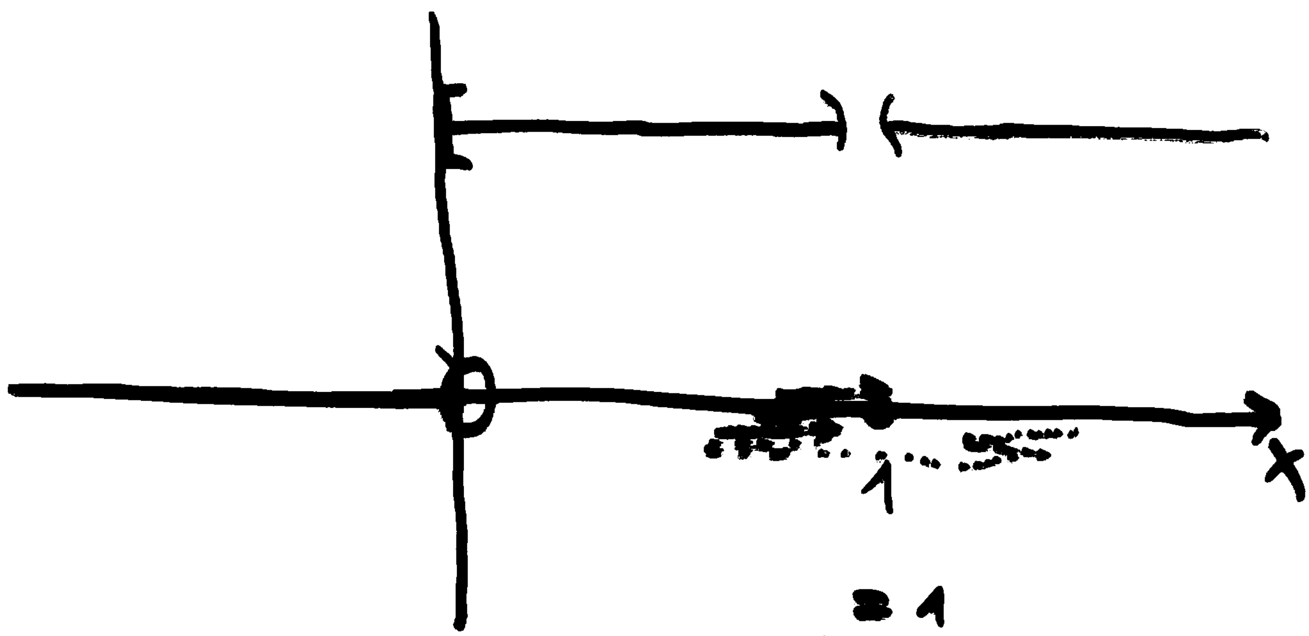
$\rightarrow \quad e^{2x} = \frac{1+y}{1-y} \rightarrow x = \frac{1}{2} \ln \frac{1+y}{1-y}$

$\rightarrow \operatorname{artanh}(x) = \frac{1}{2} \cancel{\ln \frac{1+x}{1+x}} \ln \frac{1+x}{1-x}$

Grenzwerte von Funktionen

Bsp:

$$f(x) := \begin{cases} 0 \,, & x < 0 \text{ oder } x = 1 \\ 1 \,, & \text{sonst} \end{cases}$$



$$\lim_{x \to 1} f(x) = \lim_{x_n \to 1} \overbrace{f(x_n)}^{=1} = 1 = g$$

$$\lim_{n \to \infty} x_n = 1 \qquad \neq f(1)$$

$$g \neq f(1)$$

obwohl $\lim x_n = 1$