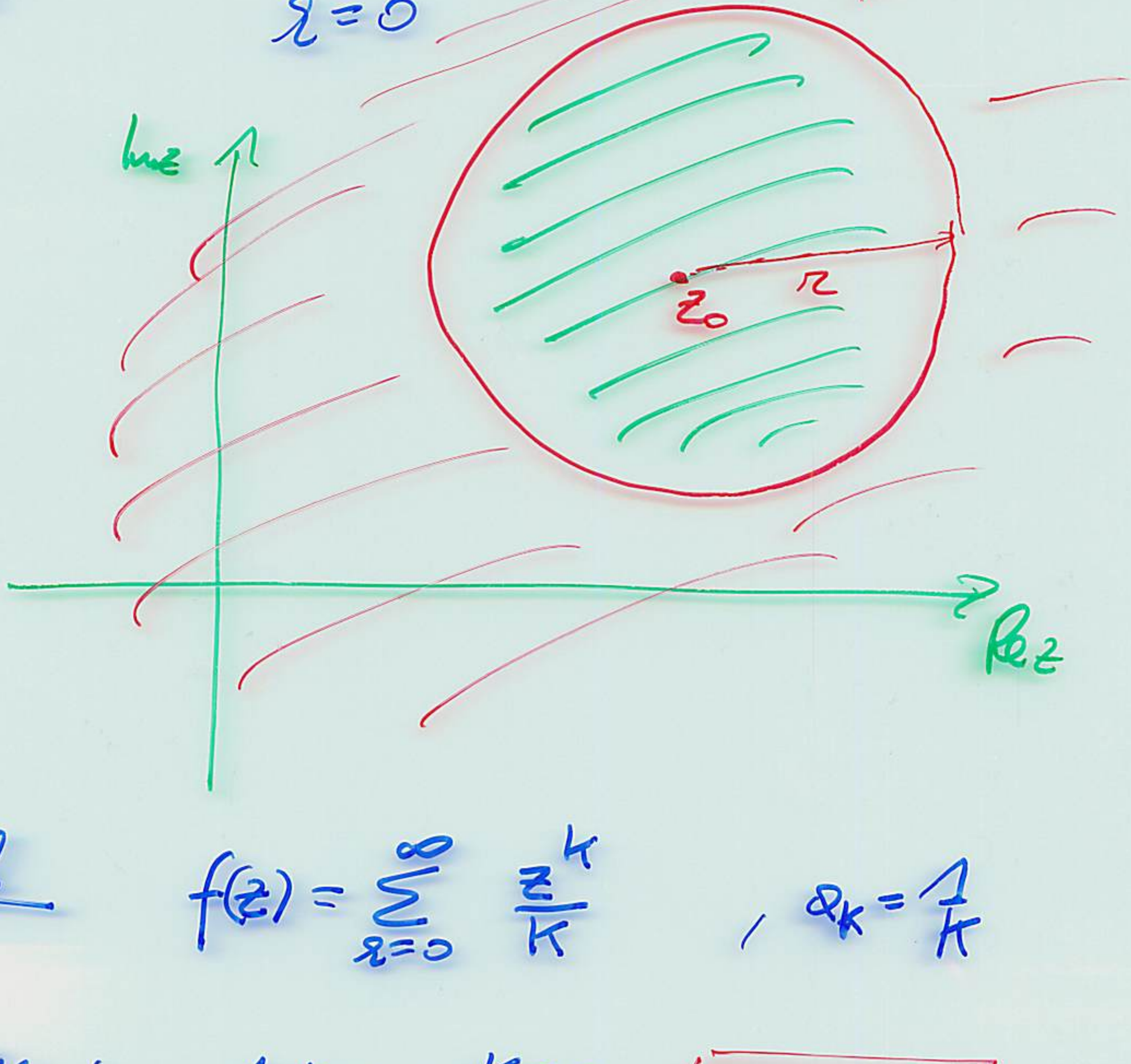
$\cancel{f(z)}$ $\cancel{f(z)}$

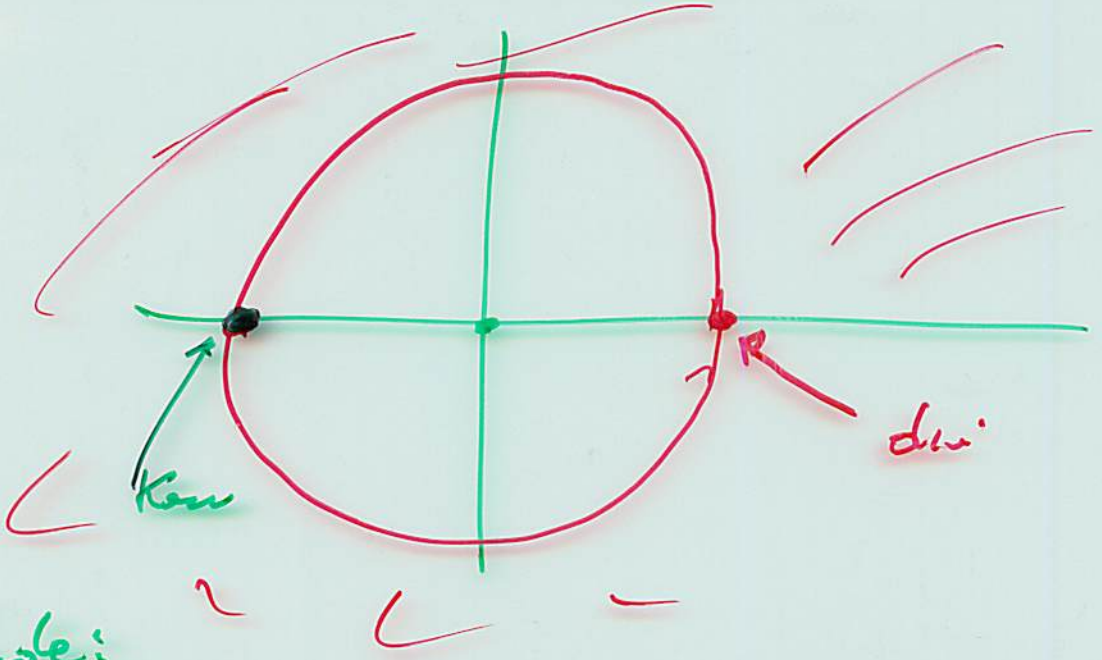
$$f(z) = \sum_{k=0}^{\infty} a_k (z-z_0)^k$$



<u>Bspl</u>

$$f(z) = \sum_{k=0}^{\infty} \frac{z^k}{k} \quad , \quad a_k = \frac{1}{k}$$

$$\left|\frac{a_k}{a_{k+1}}\right| = \frac{1}{k}\,\frac{k+1}{1} \xrightarrow{k\to\infty} \boxed{1 = r}$$



Rand:

z.B. $z=1$ $\quad f(1) = \sum_{k=0}^{\infty} \frac{1}{k}$ $\quad$ div

$z=-1$ $\quad f(-1) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k}$ $\quad$ Konv

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$$f(z) = \sum_{k=0}^{\infty} a_k (z-z_0)^k$$

für $|z-z_0| < R$ Konv., d.h.

$$S_N(z) = \sum_{k=0}^{N} a_k (z-z_0)^k \longrightarrow S(z) = f(z)$$

$$S_N'(z) = \sum_{k=0}^{N} a_k\, k\, (z-z_0)^{k-1} \longrightarrow \tilde{S}(z)$$

Ergeb.

$$\boxed{f'(z) = \tilde{S}(z)}$$

Bspl: geom. Reihe

$$f(z) = \sum_{k=0}^{\infty} z^k \quad / \quad a_k = 1$$

$$R = \lim_{k\to\infty} \left| \frac{a_k}{a_{k+1}} \right| = \lim_{k\to\infty} 1 = 1$$

$$f(z) = \frac{1}{1-z}$$

Bspl: $\exp(z) = \sum_{k=0}^{\infty} \frac{z^k}{k!}$

$$\left| \frac{a_k}{a_{k+1}} \right| = \frac{(k+1)!}{k!} = k+1 \longrightarrow \infty$$

ZVo 4

$$\int z^n \, dz \;\hat{=}\; \frac{z^{n+1}}{n+1}$$

$$z^n = \left(\frac{z^{n+1}}{n+1}\right)'$$

$$f(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z_0 = 0$$

$$g(z) = \sum_{k=0}^{\infty} b_k z^k \qquad z_0 = 0$$

$$f(z)\cdot g(z) = \left(a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \dots\right)$$

$$\cdot\left(b_0 + b_1 z + b_2 z^2 + b_3 z^3 + \dots\right)$$

$$= a_0 b_0 + (a_0 b_1 + b_0 a_1) z + (a_0 b_2 + a_1 b_1 + a_2 b_0) z^2$$

$$+ \dots$$

$$f(z) = \frac{1}{1-z} = \sum_{l=0}^{\infty} z^{l}$$

$$g(z) = \exp(z) = \sum_{k=0}^{\infty} \frac{z^{k}}{k!}$$

$$f(g(z)) = \frac{1}{1-g(z)} = \frac{1}{1-\exp(z)} =$$

$$= 1 + g(z) + g(z)^2 + g(z)^3 + \ldots$$

$$= 1 + \left(1 + z + \frac{z^2}{2} + \frac{z^3}{3!} \ldots\right) + \left(1 + z + \frac{z^2}{2} + \ldots\right)^2 + \ldots$$

$$= 1 + \left(1 + z + \frac{z^2}{2} + \frac{z^3}{3}\right) + \underbrace{\left(1 + 2z + \left(\frac{1}{2} + 1 + \frac{1}{2}\right) z^2 \ldots\right)}_{g(z)^2}$$

$$= (1 + 1 + 1 +) + z(1 + 2 + \ldots)$$

$$+ z^2\left(\frac{1}{2} + (\quad) + \ldots\right)$$

dw:

$$f(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z_0 = 0$$

$$f(0) \neq 0$$

$$\frac{1}{f(z)} = \sum_{k=0}^{\infty} d_k z^k$$

$$1 = f(z)\,\frac{1}{f(z)} = \left(a_0 + a_1 z + a_2 z^2 + \dots\right)$$

$$\cdot \left(d_0 + d_1 z + d_2 z^2 + \dots\right.$$

$$= \underbrace{a_0 d_0}_{=1} + \underbrace{\left(a_1 d_0 + a_0 d_1\right)}_{=0} z + \underbrace{\left(a_0 d_2 + a_1 d_1 + a_2 d_0\right)}_{=0} z^2$$

gegeben a_K

gesucht d_K

$\Rightarrow \quad d_0 a_0 = 1 \qquad \Rightarrow d_0$

$a_1 d_0 + a_0 d_1 = 0 \qquad \Rightarrow d_1$

$a_0 d_2 + a_1 d_1 + a_2 d_0 = 0 \qquad \Rightarrow d_2$