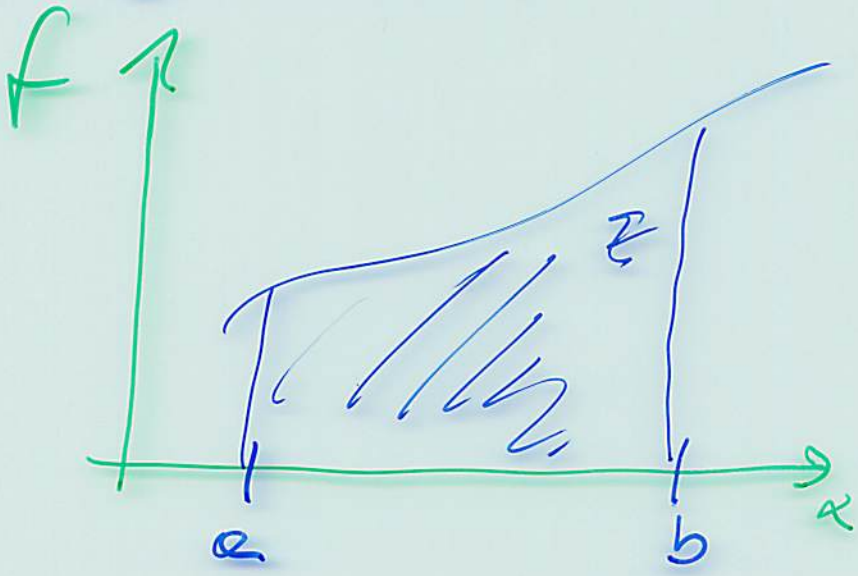
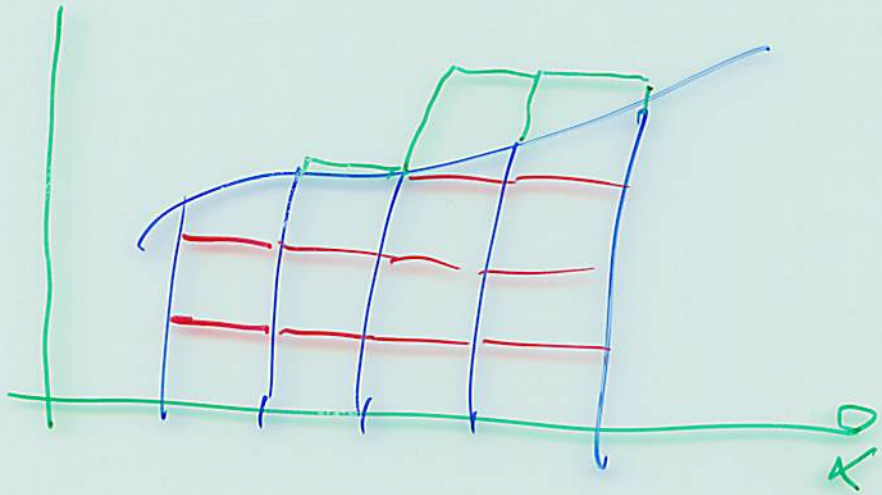
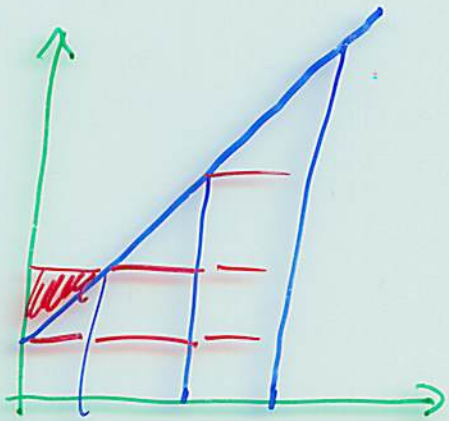
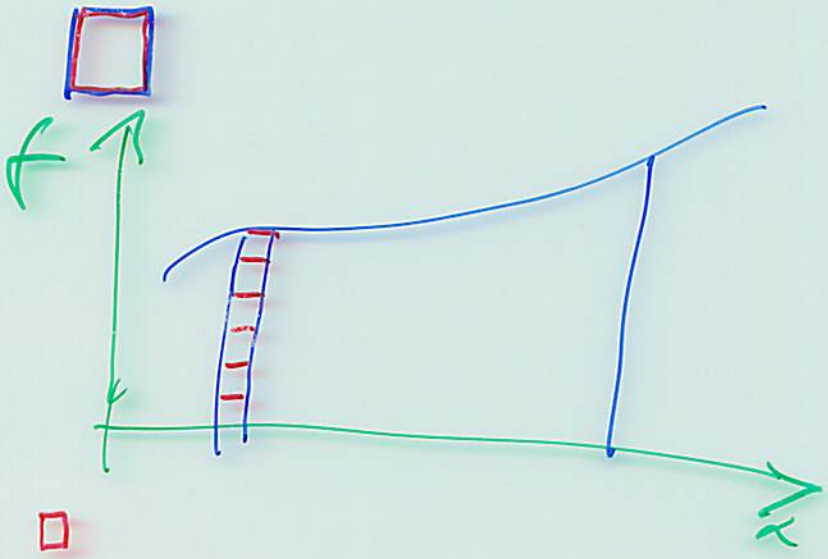
Integration

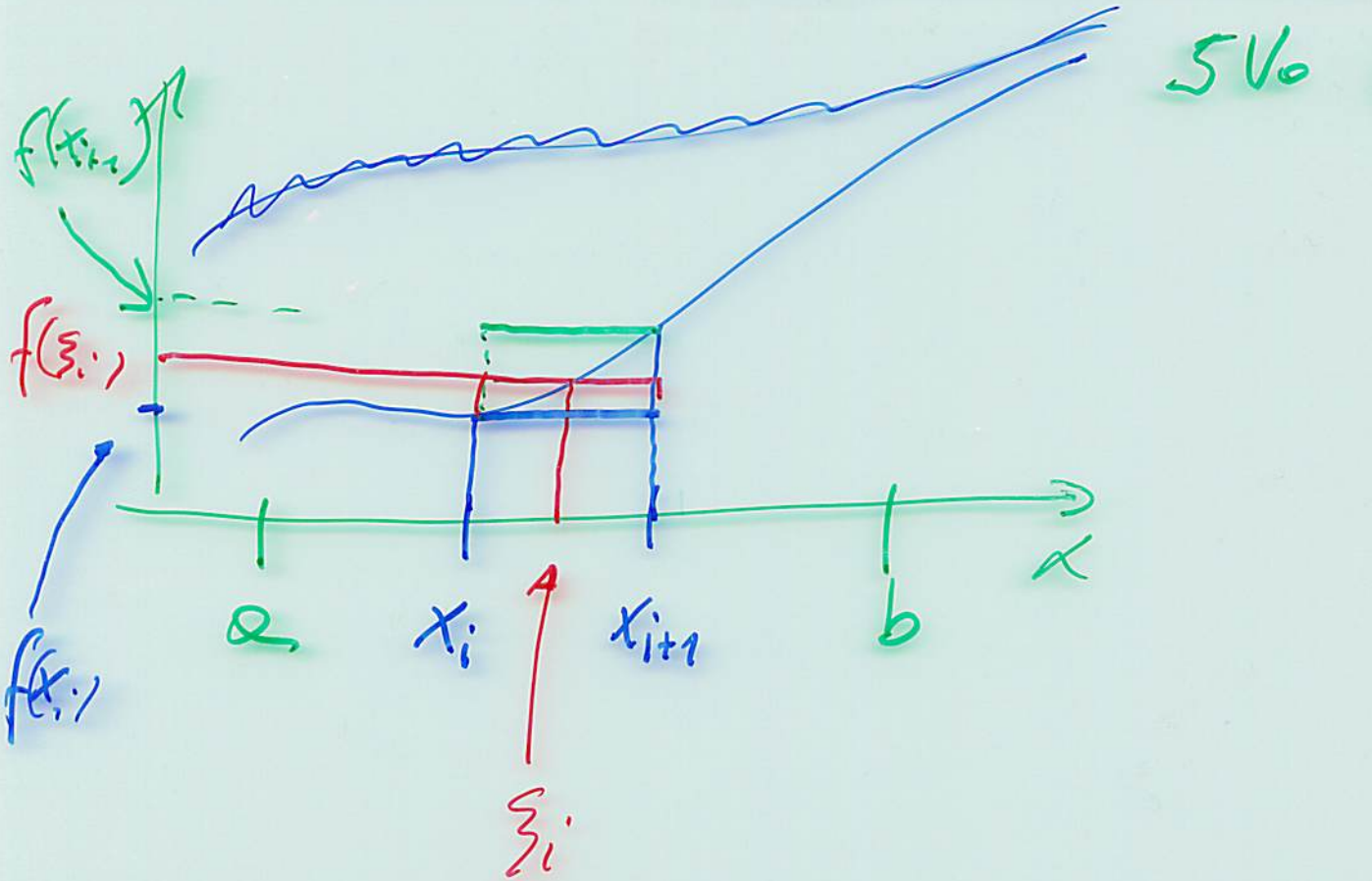
5V0 (1)

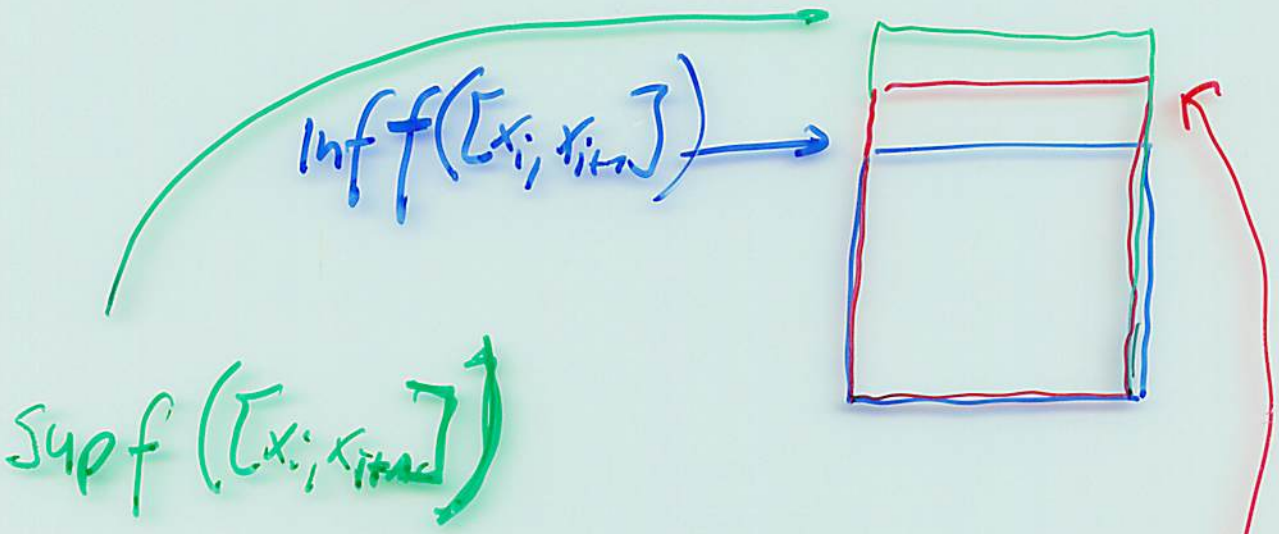


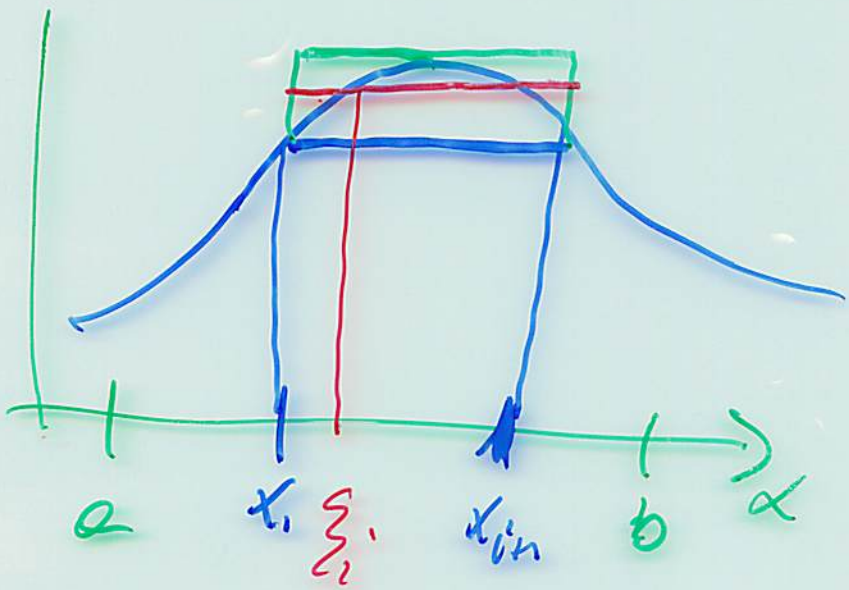


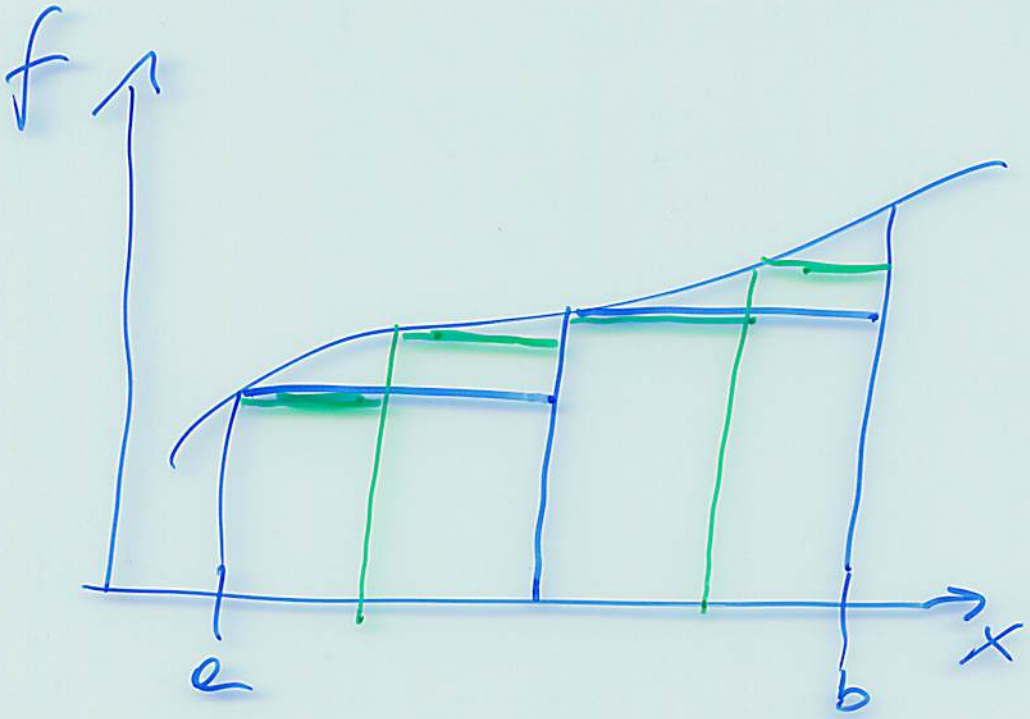





f(x_{i+1})
f(ξ_i)
f(x_i)
a
x_i
x_{i+1}
b
x
ξ_i


inf f([x_i; x_{i+1}])
sup f([x_i; x_{i+1}])
f(ξ_i)


a
x_i
ξ_i
x_{i+1}
b
x

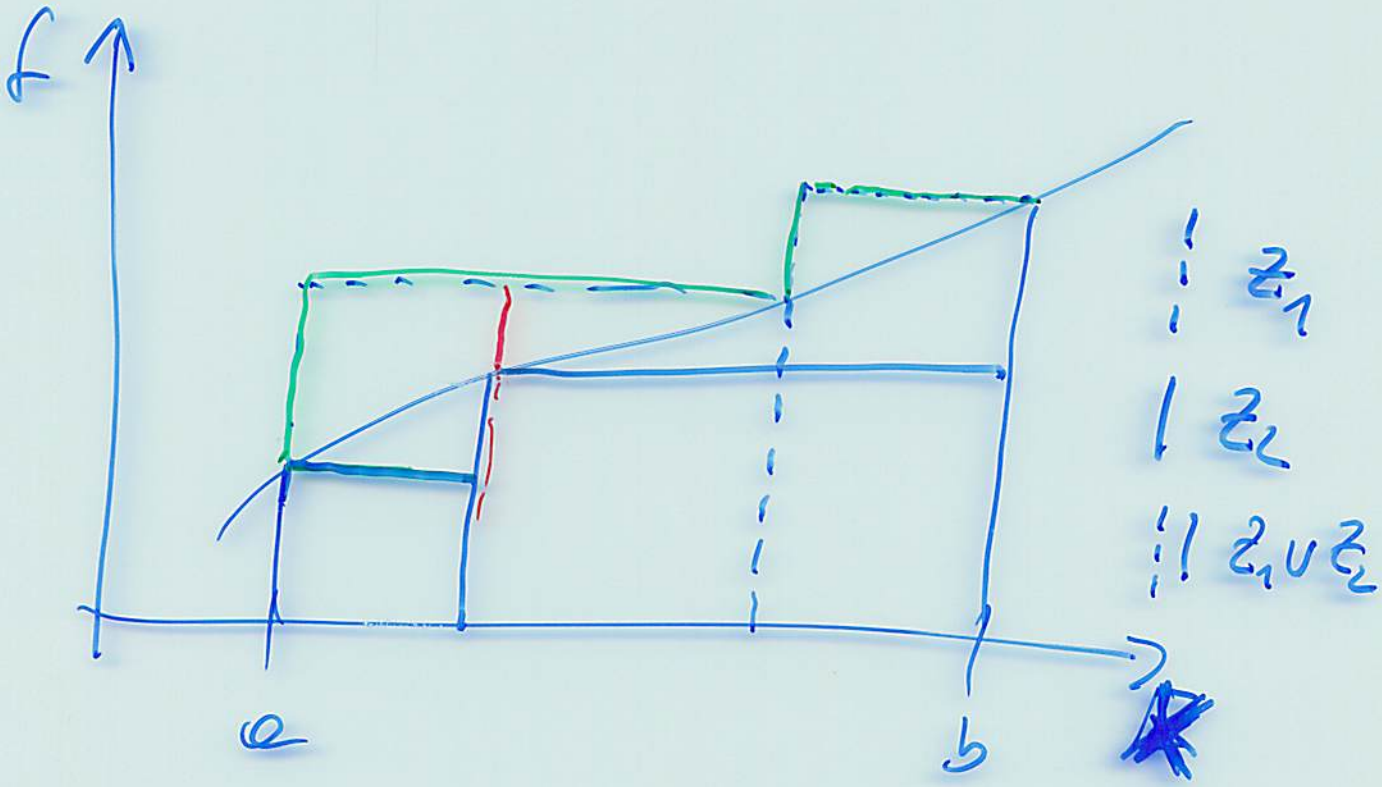


z_2

z_1

$z_1 \supset z_2$

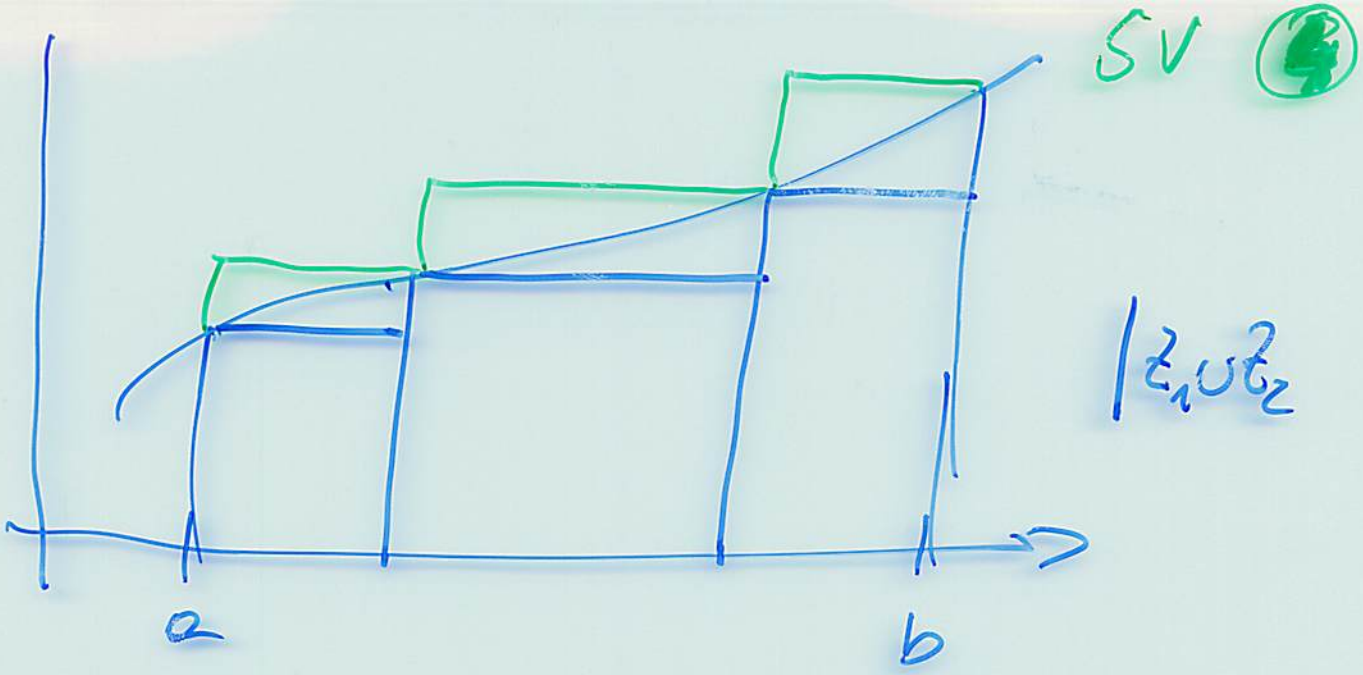
$$U_f(z_1) \geq U_f(z_2)$$



z_1

z_2

$z_1 \cup z_2$



$$L_f(z_1) \leq L_f(z_1 \cup z_2) \leq U_f(z_1 \cup z_2) \leq U_f(z_1)$$

$$L_f(z) \leq U_f(z_0) \quad \forall z$$

$$L_f(z_0) \leq U_f(z) \quad \forall z$$

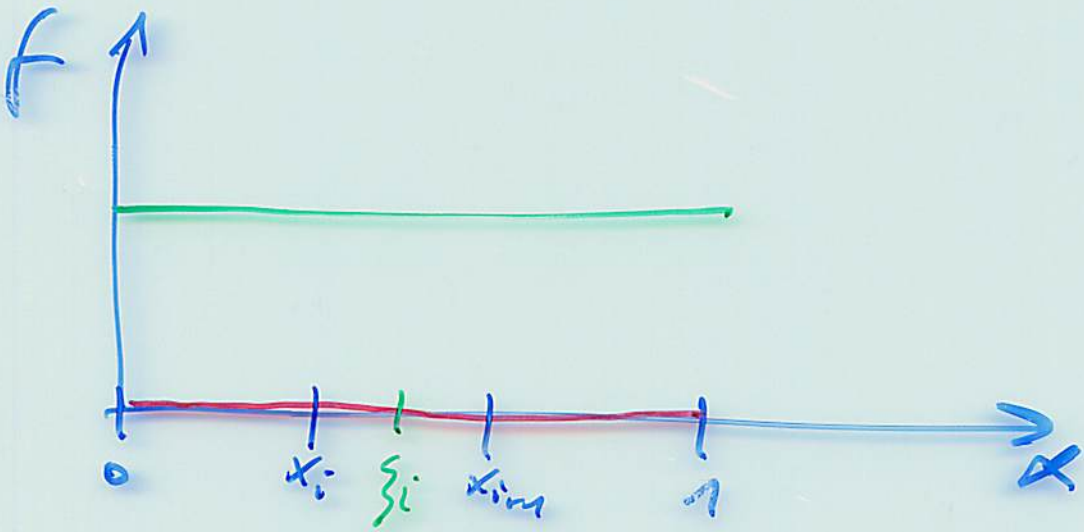
$$U_f(Z_n) = \sum_{i=0}^{n-1} \frac{i}{n} \left(\frac{i+1}{n} - \frac{i}{n} \right) =$$

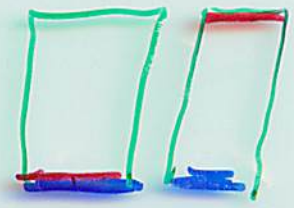
$\frac{i}{n}$: $\inf f([x_i, x_{i+1}])$; $\frac{i+1}{n}$: x_{i+1}; $\frac{i}{n}$: x_i

$$= \frac{1}{n^2} \sum_{i=0}^{n-1} i \cdot 1 = \frac{1}{n^2} n \cdot \frac{n-1}{2} =$$

$$= \frac{1}{2} - \frac{1}{2n}$$

$$f(x) = \begin{cases} 0 & x \in [0,1] \cap \mathbb{Q} \\ 1 & x \in [0,1] \setminus \mathbb{Q} \end{cases}$$

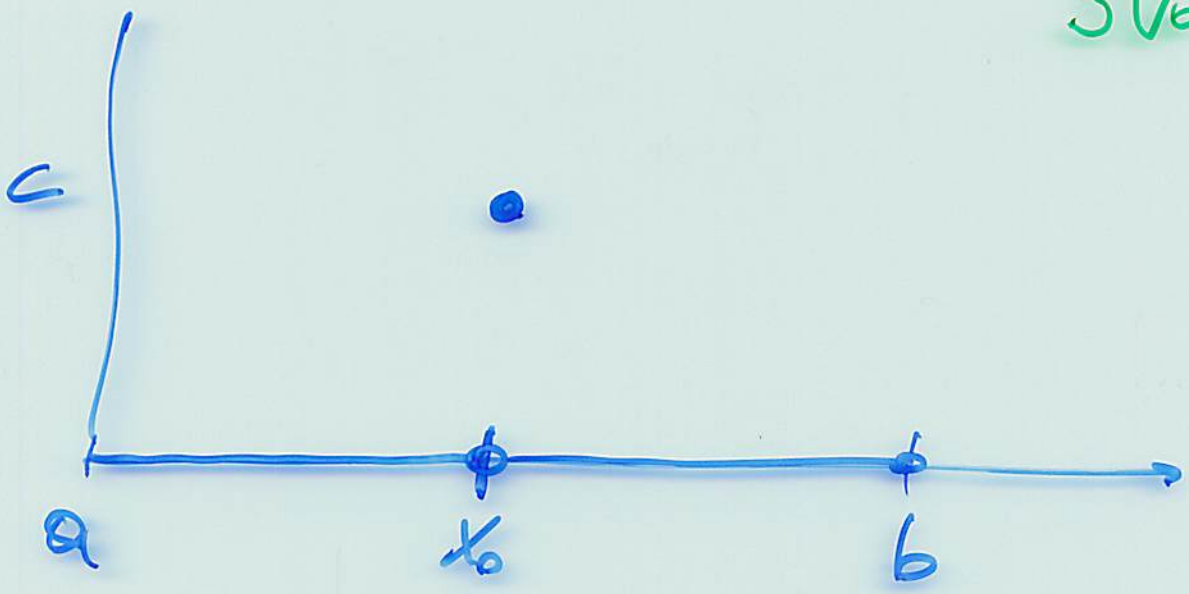




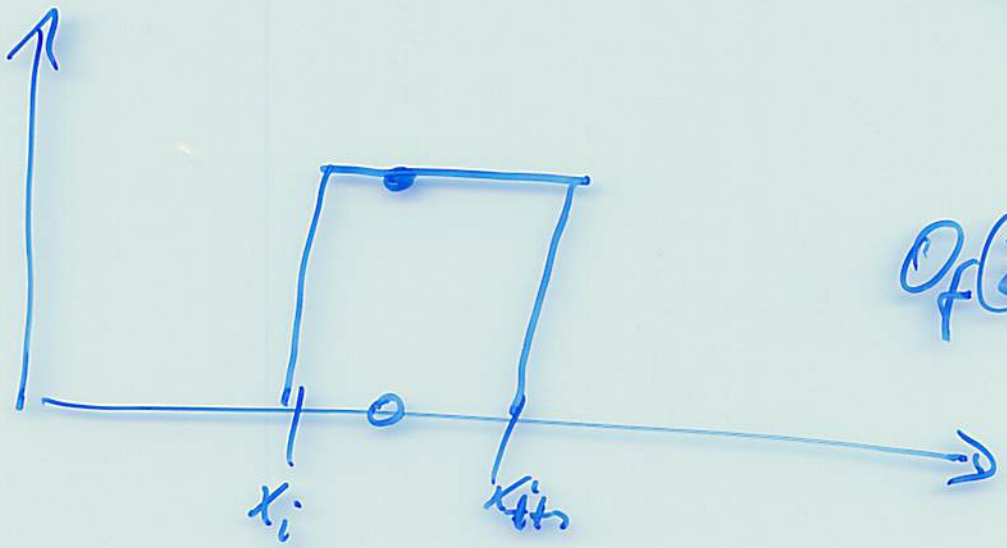
$\inf f([x_i, x_{i+1}]) = 0$

$\sup f([x_i, x_{i+1}]) = 1$

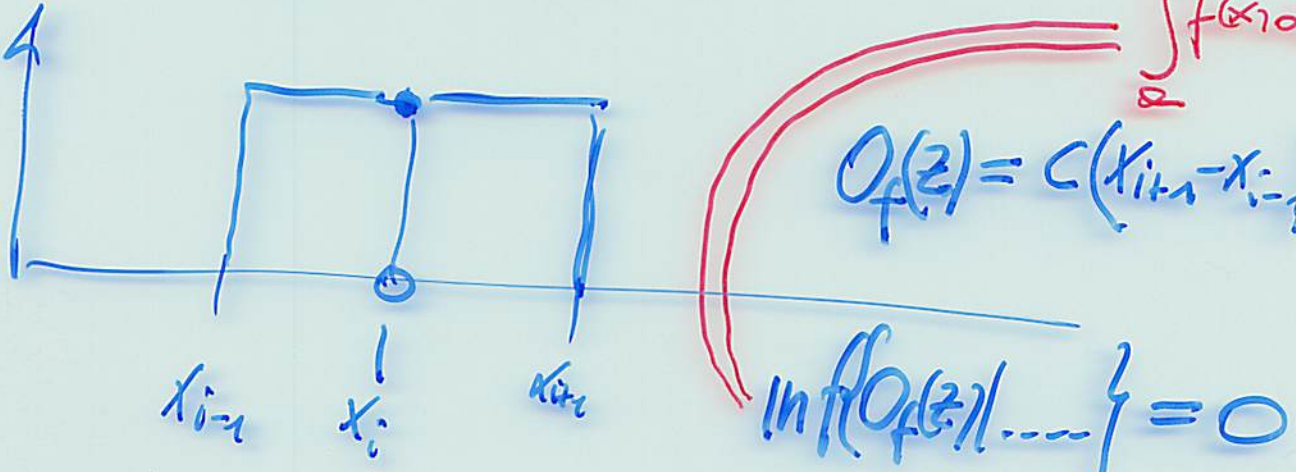
5V6 (6)



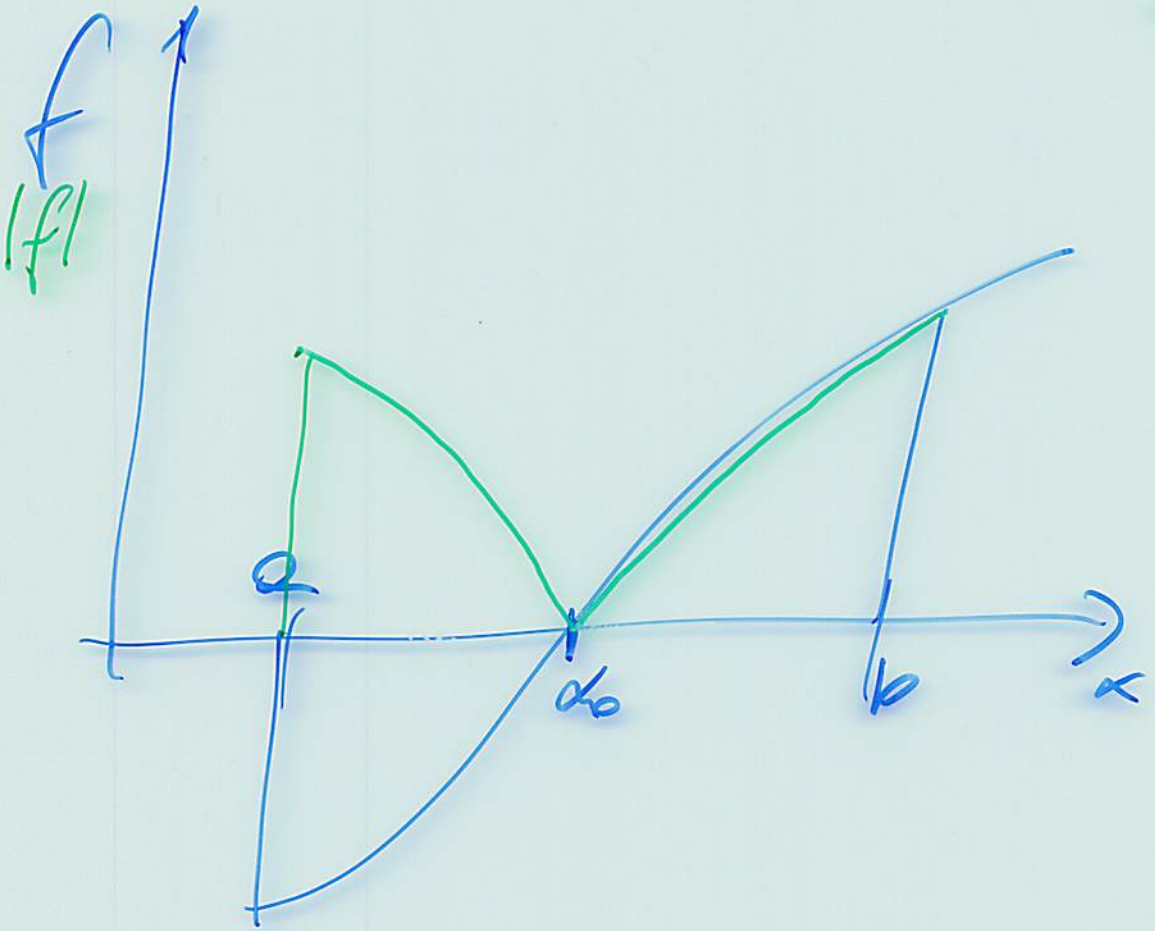
$$\inf f([x_i, x_{i+1}]) = 0 \quad \Rightarrow \quad \int_a^b f(x)\,dx = 0$$



$$O_f(Z) = c(x_{i+1} - x_i)$$



5V. (7)



$$\int_a^{x_0} f(x)\,dx + \int_{x_0}^{b} f(x)\,dx = \int_a^b f(x)\,dx$$

≤ 0 ≥ 0

$$\int_a^{x_0} |f(x)|\,dx + \int_{x_0}^{b} |f(x)|\,dx = \int_a^b |f(x)|\,dx$$

$$= \int_{x_0}^{b} f(x)\,dx$$

$$- \int_a^{x_0} f(x)\,dx$$

≥ 0

SVo (5)

Integr. Fkt

∪ ?

stetige Fkt.

∪

differb. Fkt

∪

2 differb. Fkt

1dimens.