Guten Morgen!

7V6 1

$f : [a,b] \to \mathbb{R}$ stetig

$$F(x) = \int_a^x f(t)\,dt$$ ist Stammfkt von f

(d.h. $F'(x) = f(x)$)

$$\Rightarrow \quad \int_a^b f(t)\,dt = F(b) - F(a)$$

Hauptsatz

$$F(x) = \int f(x)\,dx$$

Produktregel

$$\int uv'\,dx = uv - \int u'v\,dx$$

$$\int_a^b f(h(t))\,h'(t)\,dt = \int_{h(a)}^{h(b)} f(x)\,dx$$

$$= F(h(b)) - F(h(a))$$

$$(F' = f)$$

Mittelwertsatz

einfache Version:

$\exists\, \xi \in [a,b]$

$$\int_a^b f(x)\,dx = f(\xi)\,(b-a)$$

Bspl: $$\int \frac{1}{1+x^2}\,dx = \arctan x$$

Bspl: $$\int \frac{2x}{(1+x^2)^2}\,dx = \left[\begin{array}{l} 1+x^2 = u \\ 2x\,dx = du \end{array}\right] = (*)$$

(4c) $= \int \frac{du}{u^l} = \int u^{-l}\,du =$ 7b ③

$= \frac{1}{(1-l)} u^{-l+1} + C =$

$= \frac{1}{1-l} \frac{1}{u^{l-1}} + C$

Problem: $\frac{1}{x^2-x}$

allg $\frac{p(x)}{q(x)}$ p, q Polynome

Bspl $\frac{x^2}{x-1} = \frac{x^2-1+1}{x-1} = x+1+\frac{1}{x-1}$

1. Polynomdivision

$$\int \frac{x^2}{x-1}\,dx = \frac{x^2}{2} + x + \ln|x-1| + C$$

2. Nenner zerlegen

7Vo 5

$$\frac{1}{x^2-x} = \frac{1}{x(x-1)}$$

3. Partialbruchzerlegung

Ansatz

$$\frac{1}{x(x-1)} = \frac{a}{x} + \frac{b}{x-1} = (*)$$

Suche a, b

$$1 = a(x-1) + bx$$

Koeffizientenvergl.

$$1 = -a$$

$$0 = a + b \qquad b = 1$$

$$(*) = \frac{-1}{x} + \frac{1}{x-1}$$

$$\int \frac{1}{x^2-x}\,dx = \int \frac{1}{x(x-1)}\,dx = \int\left(\frac{-1}{x} + \frac{1}{x-1}\right)dx$$

$$= -\ln|x| + \ln|x-1| + C$$

$$= \ln\left|\frac{x-1}{x}\right| + C$$

4V0 5

$$\frac{1}{x^3-x^2} = \frac{1}{x^2(x-1)}$$

$$= \frac{a_1}{x} + \frac{a_2}{x^2} + \frac{b}{x-1}$$

... $1 = a_1 x(x-1) + a_2(x-1) + b x^2$

$\Rightarrow$ $1 = -a_2$ $\quad a_1 = -1$

$0 = -a_1 + a_2$ $\quad a_2 = -1$

$0 = a_1 + b$ $\quad b = +1$

Falsch:

$$\frac{1}{x^2(x-1)} = \frac{a_1}{x} + \frac{b}{x-1}$$

$$1 = a_1 x(x-1) + b x^2$$

$1 = 0$

$0 = -a_1$

$0 = a_1 + b$

$1 = -a_1$

$0 = a_1$

$0 = b$

Falsch

$$\frac{1}{x^2(x-1)} = \frac{a_1}{x^2} + \frac{b}{x-1}$$

... $1 = a_1(x-1) + b x^2$

Komplexe NS (reelle Polynome) FVO ⑥

$x_0 = a + ib$ NS

$\bar{x}_0 = a - ib$ auch NS

$$\left(x-(a+ib)\right)\left(x-(a-ib)\right) =$$

$$= \left(x-a-ib\right)\left(x-a+ib\right)$$

$$= (x-a)^2 + b^2$$

$\Rightarrow$ i.a.

$$\frac{1}{\left[(x-a)^2+b^2\right]^e}$$

Bsp

$$\frac{1}{x(x^2+b^2)} = \frac{a}{x} + \frac{b_1x+c_1}{x^2+b^2}$$

$$\frac{1}{x^2(x^2+b^2)^2} = \frac{a_1}{x} + \frac{a_2}{x^2} + \frac{b_1x+c_1}{(x^2+b^2)} + \frac{b_2x+c_2}{(x^2+b^2)^2}$$

Fazit

Iterationsformel (Ie)

nur notwendig, falls

mehrfache Komplexe NS

$$R(x) = \frac{P(x)}{Q(x)}$$

$$R(e^x)$$

$$\int R(e^x)\,dx = \left[\begin{array}{l} e^x = u \\ e^x dx = du \quad dx = \frac{du}{e^x} = \frac{du}{u} \end{array}\right]$$

$$= \int \frac{R(u)}{u}\,du$$

Bspl.

$$\int \frac{e^x - 1}{e^x}\,dx = \int \frac{u-1}{u} \cdot \frac{1}{u}\,du =$$

$$= \int \frac{u-1}{u^2}\,du = \int \frac{1}{u} - \int \frac{1}{u^2}$$

$$\int R(\cos x, \sin x)\,dx = \int R\left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}\right)\underline{\frac{2}{1+t^2}}\,dt$$

?

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\tan x = \frac{2t}{1-t^2}$$

$$x = \arctan \frac{2t}{1-t^2}$$

$$\frac{dx}{dt} = \frac{1}{1+\left(\frac{2t}{1-t^2}\right)^2} \cdot \frac{2(1-t^2)+(2t)^2}{(1-t^2)^2} =$$

$$= \frac{2(1+t^2)}{(1-t^2)^2+4t^2}$$

$$= \frac{2(1+t^2)}{1-2t^2+t^4+4t^2} = \frac{2\cancel{(1+t^2)}}{(1+t^2)^{\cancel{2}}}$$

$f(x) = \frac{1}{x^2}$

$f: (0,\infty) \to \mathbb{R}$

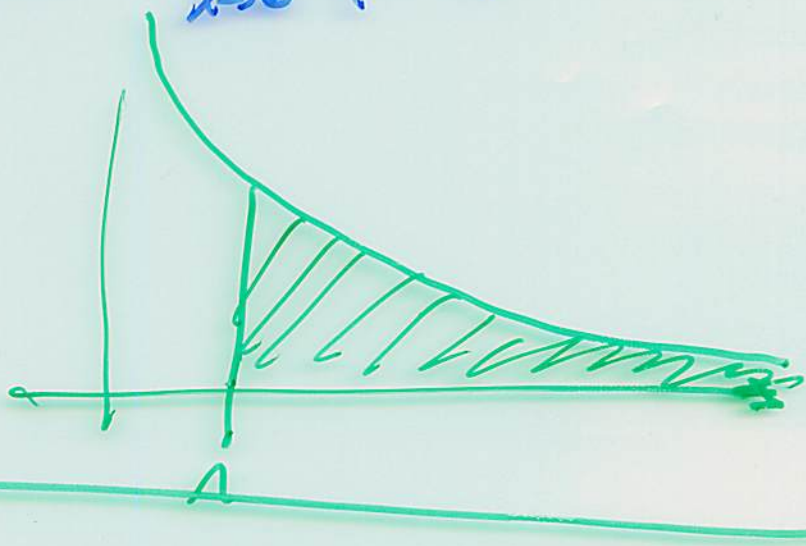
lokal iwb $\quad \forall\, [a,b] \subset (0,\infty)$

$\exists \int_a^b \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_a^b$

$= \frac{1}{a} - \frac{1}{b}$

$$\int_1^x f(t)\,dt = 1 - \frac{1}{x}$$

$$\int_1^\infty f(t)\,dt = \lim_{x\to\infty} \int_1^x f(t)\,dt = \lim_{x\to\infty}\left(1 - \frac{1}{x}\right) = 1$$



$f(x) = \frac{1}{x^2}$ $\qquad f:(0,\infty) \to \mathbb{R}$

$$\int_0^1 \frac{1}{x^2}\,dx = \lim_{x \to 0+} \int_x^1 \frac{1}{t^2}\,dt =$$

$$= \lim_{x \to 0+} -\frac{1}{t}\Big|_x^1 =$$

$$= \lim_{x \to 0+} \left(\frac{1}{x} - 1\right) \qquad \nexists$$

$g(x) = \frac{1}{\sqrt{x}}$

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{x \to 0+} \int_x^1 \frac{1}{\sqrt{t}}\,dt =$$

$$= \lim_{x \to 0+} 2\sqrt{t}\Big|_x^1 = \lim_{x \to 0+} \left(2 - 2\sqrt{x}\right) = 2$$

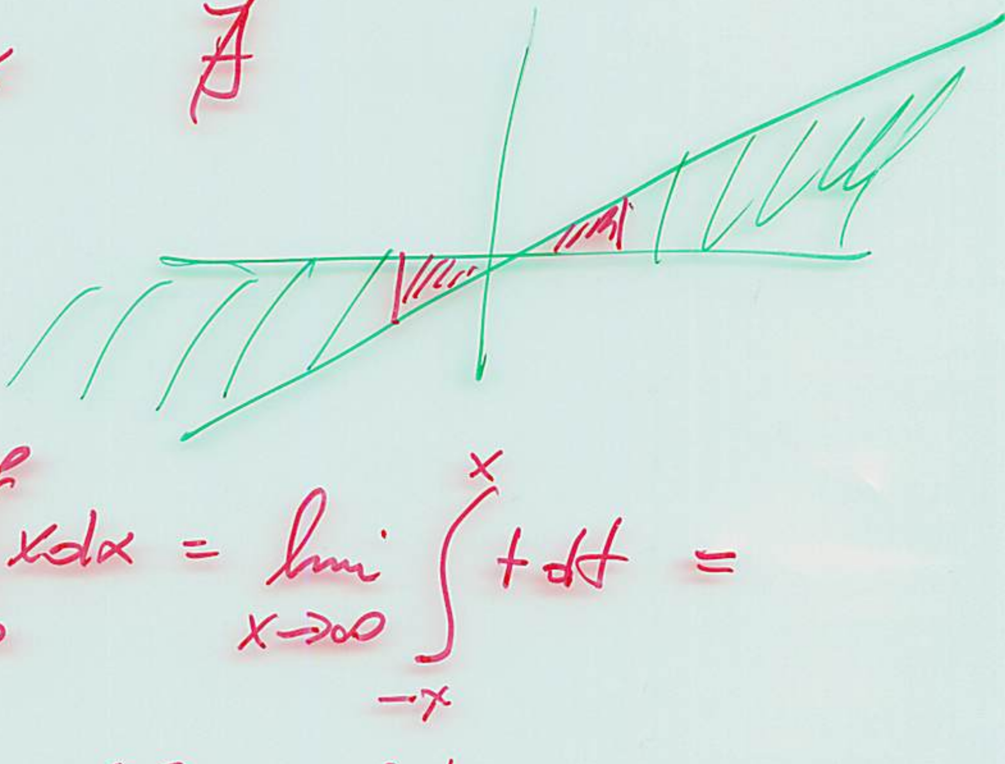
$f(x) = x$

$f: \mathbb{R} \to \mathbb{R}$ lokal int.

$$\int_0^\infty x\,dx = \lim_{x\to\infty}\int_0^x t\,dt = \lim_{x\to\infty}\left(\frac{x^2}{2}\right) \nexists$$

$$\int_{-\infty}^0 x\,dx = \lim_{x\to-\infty}\int_x^0 t\,dt = \lim_{x\to-\infty}\left(-\frac{x^2}{2}\right) \nexists$$

$$\Rightarrow \int_{-\infty}^{\infty} x\,dx \quad \nexists$$



<u>Aber</u>

$$CHW\int_{-\infty}^{\infty} x\,dx = \lim_{x\to\infty}\int_{-x}^{x} t\,dt =$$

$$= \lim_{x\to\infty}\left(\frac{x^2}{2} - \frac{(-x)^2}{2}\right) = \lim_{x\to\infty} 0 = 0$$

$$\exists$$