uneigentliche I.

$$\int_a^\infty f(x)\,dx \quad ?$$

Falls $\lim\limits_{b\to\infty}\int_a^b f(x)\,dx \quad \exists$

$$= \int_a^\infty f(x)\,dx$$

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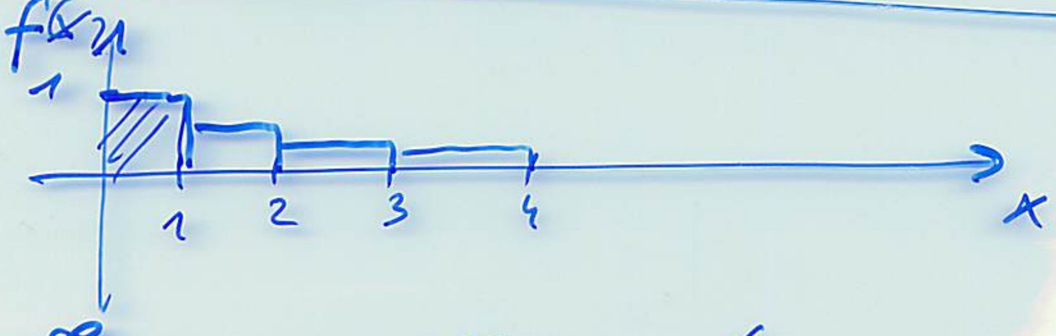
§ falls $f(a) \nexists$

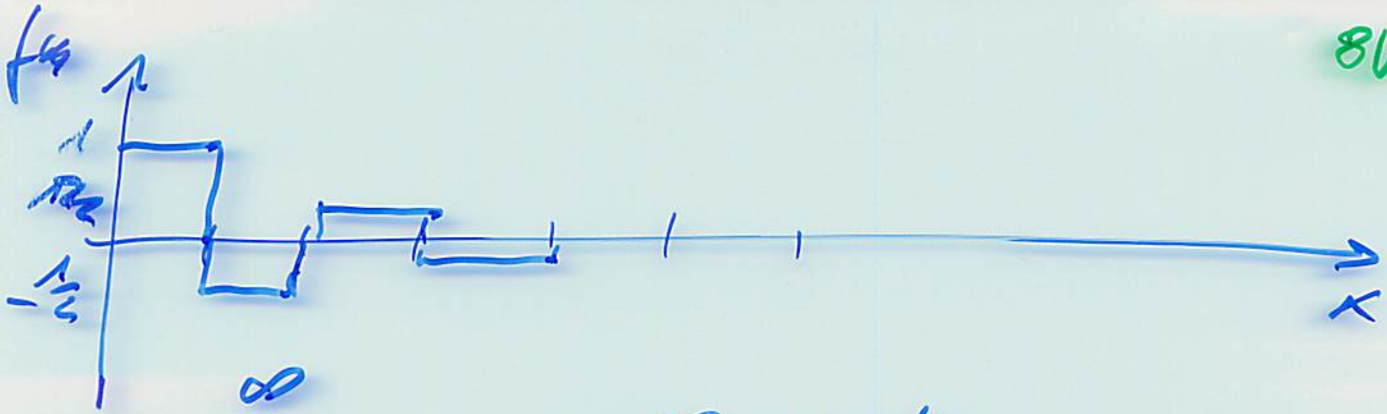
$$\int_a^b f(x)\,dx$$

$$\parallel$$

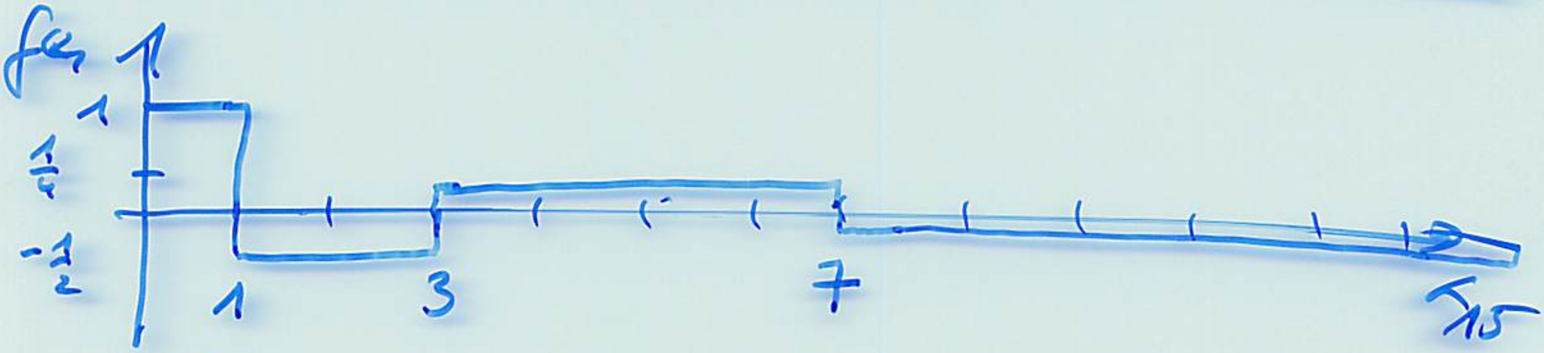
$$\lim_{\varepsilon\to 0}\int_{a+\varepsilon}^b f(x)\,dx \quad \exists$$



$$\int_0^\infty f(x)\,dx = \sum_{n=1}^\infty \frac{1}{n} \quad \nexists$$



$$\int_0^\infty f(x)\,dx = \sum_{n=1}^\infty \frac{(-1)^n}{n} \quad \exists$$



1

$1 - 1 = 0$

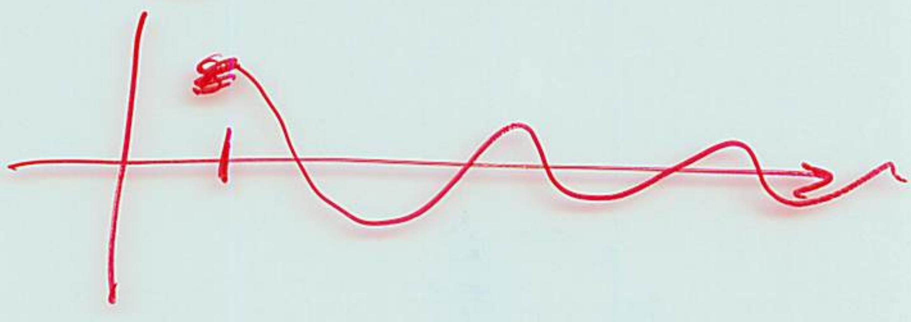
$1 - 1 + 1 = 1$

$1 - 1 + 1 - 1 = 0$

$$\Rightarrow \int_0^\infty f(x)\,dx \quad \exists$$

$$\int_1^\infty \frac{\sin x}{x}\,dx$$

$$\int_1^\infty \frac{1}{x}\,dx \quad \nexists$$



Kriterien

8VO ③

$$\int_a^\infty f(x)\,dx \quad \exists \quad , \quad \text{d.h.}$$

$$\lim_{b\to\infty} F(b) = \lim_{b\to\infty} \int_a^b f(x)\,dx \overset{\exists}{=} F$$

d.h. $\forall (b_n)_{n\in\mathbb{N}}$, $b_n \to \infty$, $\lim_{n\to\infty} F(b_n) = F$

$\forall \varepsilon > 0 \;\; \exists N = N(\varepsilon)$, $|F(b_n) - F| < \varepsilon \quad \forall n \geq N(\varepsilon)$

oder $\forall \varepsilon > 0 \;\; \exists N = N(\varepsilon) \quad |F(b_n) - F(b_m)| < \varepsilon$

$\forall n, m \geq N(\varepsilon)$

oder $\forall \varepsilon > 0 \;\; \exists b_{N(\varepsilon)} = C \quad |F(b_n) - F(b_m)| < \varepsilon$

$\forall b_n, b_m > C$

d.h. $|F(b_n) - F(b_m)| = \left| \int_a^{b_n} f(x)\,dx - \int_a^{b_m} f(x)\,dx \right|$

$$= \boxed{\left| \int_{b_n}^{b_m} f(x)\,dx \right| < \varepsilon}$$

$\forall b_n, b_m > C$

Bsp:

8Vo ④

$$\int_1^\infty \frac{1}{x^2}\,dx$$

$$b_n < b_m, \quad \int_{b_n}^{b_m} \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_{b_n}^{b_m} = \frac{1}{b_n} - \frac{1}{b_m} < \varepsilon$$

$$b_n > \frac{1}{\varepsilon}$$

---

also $\forall \varepsilon > 0 \quad \exists C = \frac{1}{\varepsilon} \quad \left| \int_{z_1}^{z_2} \frac{1}{x^2}\,dx \right| < \varepsilon$

$\forall z_1, z_2 > C$

---

abs Konv

$$\int_a^\infty |f(x)|\,dx \quad \exists$$

Falls $\int_a^\infty f(x)\,dx$ abs. Konv. $\Rightarrow$ Konv

$\Downarrow$ d.h.

$$\int_a^\infty |f(x)|\,dx \quad \exists$$

also $\forall \varepsilon > 0 \quad \exists C > 0 \quad \forall z_1, z_2 > C$

$$\left| \int_{z_1}^{z_2} f(x)\,dx \right| \le \int_{z_1}^{z_2} |f(x)|\,dx = \left| \int_{z_1}^{z_2} |f(x)|\,dx \right| < \varepsilon$$

$\Rightarrow$ Konv

8Vo 5

$$\int_{-\infty}^{x} \frac{e^t}{t}\,dt \qquad x<0$$

$$= \left[\begin{array}{l} t=-\tau \\ dt=-d\tau \end{array}\right] = \int_{\infty}^{-x} \frac{e^{-\tau}}{\tau}\,d\tau = \underline{-\int_{-x}^{\infty} \frac{e^{-\tau}}{\tau}\,d\tau}$$

$$f(\tau) = \frac{e^{-\tau}}{\tau}$$

$$\boxed{|f(\tau)|} = \left|\frac{e^{-\tau}}{\tau}\right| < \boxed{e^{-\tau} = g(\tau),}$$

$$\boxed{\int_{-x}^{\infty} g(\tau)\,d\tau} = \int_{-x}^{\infty} e^{-\tau}\,d\tau = -e^{-\tau}\Big|_{-x}^{\infty}$$

Majorantenkrit

$$= e^{+x} - \underbrace{e^{-\infty}}_{=0} \quad \boxed{\exists}$$

$$\int_{1}^{\infty} \frac{\sin t}{t}\,dt$$

$$|f(t)| = \left|\frac{\sin t}{t}\right| \leq \frac{1}{t}$$ aber $$\int_{1}^{\infty} \frac{1}{t}\,dt \;\nexists$$

$\cancel{0 \leq f(t)}$ Divergenzkrit. nicht anwendbar

$$\int_1^b \frac{\sin t}{t}\,dt = -\frac{\cos t}{t}\Big|_1^b - \int_1^b \frac{\cos t}{t^2}\,dt$$

direkt $b \to \infty$

$$= +\frac{\cos 1}{1} - \underbrace{\lim_{b\to\infty}\frac{\cos b}{b}}_{=0} - \lim_{b\to\infty}\int_1^b \frac{\cos t}{t^2}\,dt$$

$\exists$ weil

$$|f(t)| = \left|\frac{\cos t}{t^2}\right| \leq \frac{1}{t^2} = g(t)$$

$$\int_1^\infty g(t)\,dt = \int_1^\infty \frac{1}{t^2}\,dt \quad \exists$$

Gammafkt $\quad \Gamma(x) = \int_0^\infty e^{-t}\, t^{x-1}\,dt$

$$f(t) = e^{-t}\, t^{x-1} = \underbrace{e^{-t}\, t^{x+1}}\, t^{-2}$$

8VO 6

$$\exists t_0 > 0 \quad e^{-t} t^{x+1} < 1 \quad \forall t > t_0$$

$$\int_0^\infty e^{-t} t^{x-1}\,dt = \int_0^{t_0} e^{-t} t^{x-1}\,dt + \underbrace{\int_{t_0}^\infty e^{-t} t^{x-1}\,dt}$$

Majorantenkrit

$$\int_{t_0}^\infty e^{-t} t^{x-1}\,dt \le \int_{t_0}^\infty \frac{1}{t^2}\,dt \quad \exists$$

$$\boxed{\Gamma(x+1)} = \int_0^\infty \underbrace{e^{-t}}_{v'} \underbrace{t^x}_{u}\,dt = \underbrace{-e^{-t} t^x \Big|_0^\infty}_{=0} + x \underbrace{\int_0^\infty e^{-t} t^{x-1}\,dt}$$

$$\underbrace{-e^0 0^x}_{0} - \underbrace{e^{-\infty} \infty^x}_{=0}$$

$$= \boxed{x\,\Gamma(x)}$$

$$\boxed{\Gamma(x+1) = x\,\Gamma(x)}$$

$x = 1$ $$\boxed{\Gamma(1)} = \int_0^\infty e^{-t} t^{1-1}\,dt = -e^{-t}\Big|_0^\infty = 1$$

$\Gamma(2) = 1 \cdot \Gamma(1) = 1$

$\Gamma(3) = 2\,\Gamma(2) = 2$

$\Gamma(4) = 3\,\Gamma(3) = 3 \cdot 2 \quad \ldots \qquad \Gamma(n) = (n-1)!$

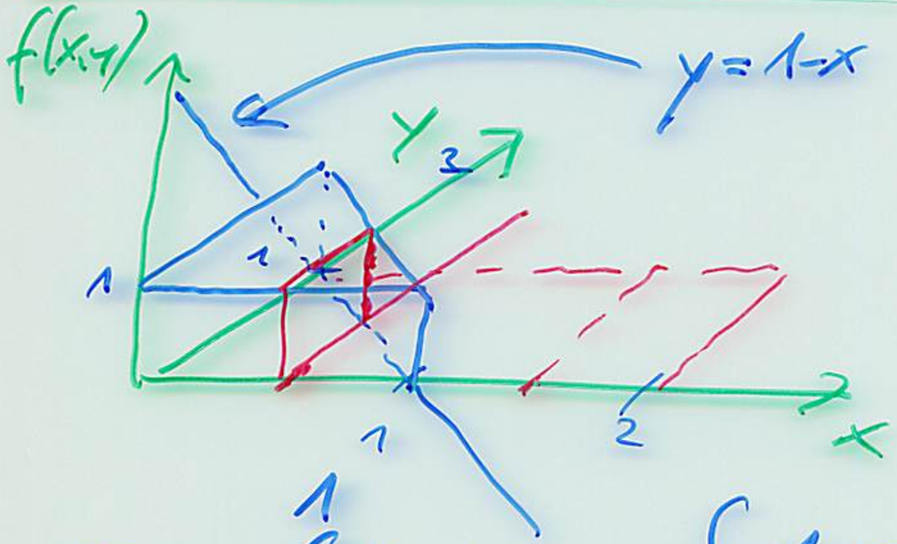
8/10 (7)

$$= \int_a^x f(t)\,dt = F(x) - F(a)$$

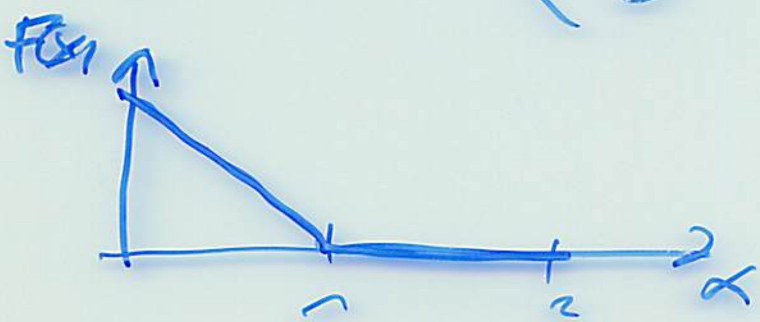
f stetig $\Rightarrow F$ differenzierbar

$\boxed{\text{H.S.DIR}}$ $F' = f$

$$F(x) = \int_a^b f(x,y)\,dy$$



$$F(x) = \int_0^1 f(x,y)\,dy = \begin{cases} 1-x & 0 \le x \le 1 \\ 0 & 1 \le x \le 2 \end{cases}$$



8 VO (8)

$$F(x) = \int_1^{\pi} \underbrace{\frac{\sin(tx)}{t}}_{\boxed{f(x,t)}} \, dt$$

stetig

$$\boxed{\frac{\partial}{\partial x} f(x,t) = \frac{\cancel{t} \cos tx}{\cancel{t}} = \cos tx}$$

also

$F'(x)$ $\exists$

$$\boxed{F'(x)} = \int_1^{\pi} \frac{\partial}{\partial x} f(x,t) \, dt = \int_1^{\pi} \cos tx \, dt =$$

$$= \frac{\sin tx}{x} \Big|_1^{\pi} = \frac{\sin \pi x}{x} - \frac{\sin x}{x}$$

$$= \boxed{\frac{\sin \pi x - \sin x}{x}}$$

$$\cancel{\int_0^a x^2 \cos x \, dx} \qquad \int_0^a t^2 \cos t \, dt = ?$$

$$F(x) = \int_0^a \cos(tx) \, dt$$

$$F'(x) = -\int_0^a t \sin(tx) \, dt$$

$$F''(x) = -\int_0^a t^2 \cos tx \, dx$$

also $\boxed{-F''(1) = \int_0^a t^2 \cos t \, dt}$

$$F(x) = \int_0^a \cos(tx)\,dt = \frac{\sin tx}{x}\Big|_0^a$$ 8V6 9

$$= \frac{\sin ax}{x}$$

$F'(x) = \ldots$

$F''(x) = \ldots$

$-F''(1) =$

$$F(x) = \int_0^\infty x\, e^{-xy}\, dy$$

$\forall x$ $$\int_0^\infty x e^{-xy}\,dy = -\frac{\cancel{x}}{\cancel{x}} e^{-xy}\Big|_0^\infty = \underline{\underline{1}}$$

$$\int_{z_1}^{z_2} x e^{-xy}\,dy = \left(e^{-xz_1} - e^{-xz_2}\right) < \varepsilon$$

$$\forall z_1 z_2 > C = -\frac{\ln\varepsilon}{x}$$

$$e^{-xz_1} < \varepsilon$$

$$\boxed{\forall x > \cancel{x_0}}$$