$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} \, dt$$

z.z. glm Konv

$$f(t) = e^{-t} t^{x-1} = \underbrace{e^{-t} t^{x+1}}_{\leq 1 \;\forall t > t_0 = t_0 \;\forall x \in [a,b]} \frac{1}{t^2}$$

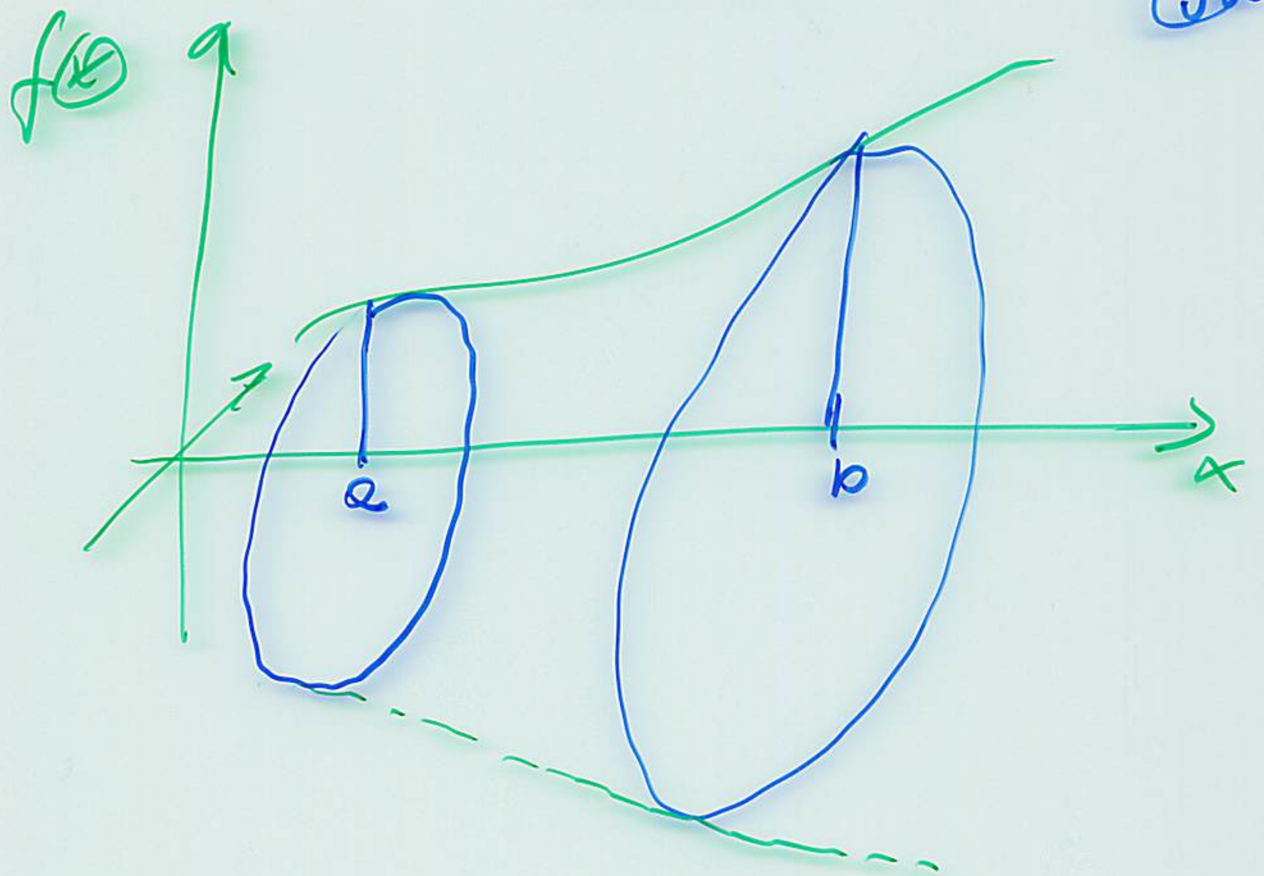
Zeige $e^{-t} t^{a+1} < 1$

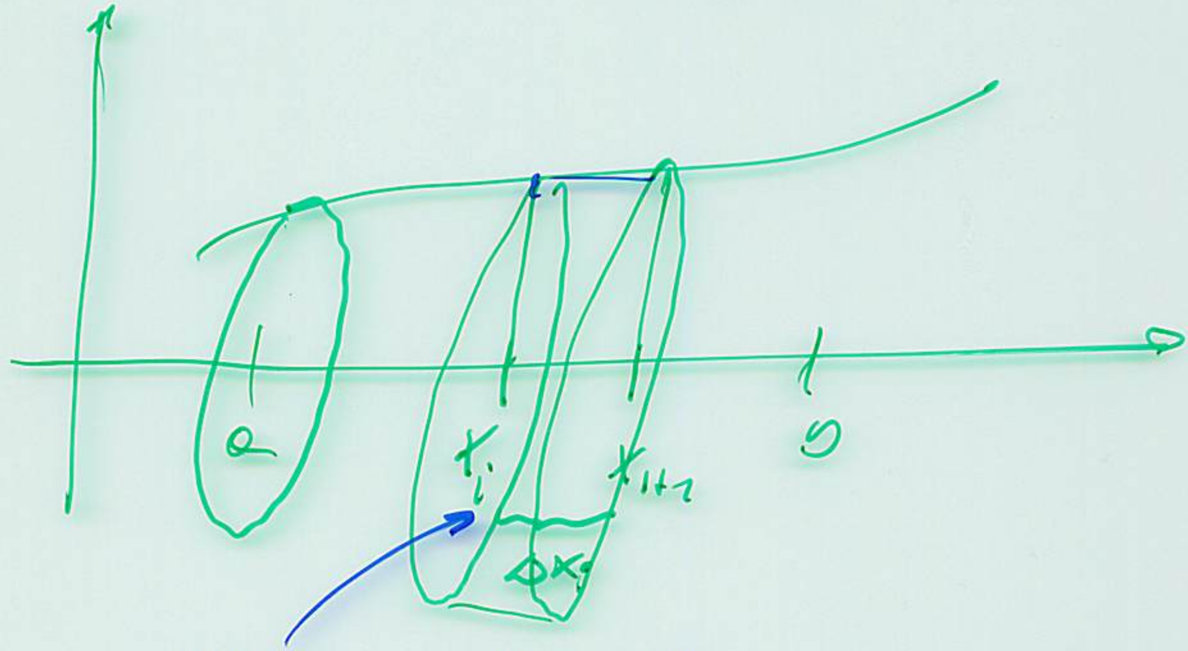
$t > t_0$

dieses t_0 ist o.k. für $x \in [a,b]$

$f_x(t) = e^{-t} t^{x-1} \ln t$ analog

$$\Rightarrow \Gamma'(x) = \int_0^\infty e^{-t} t^{x-1} \ln t \, dt$$

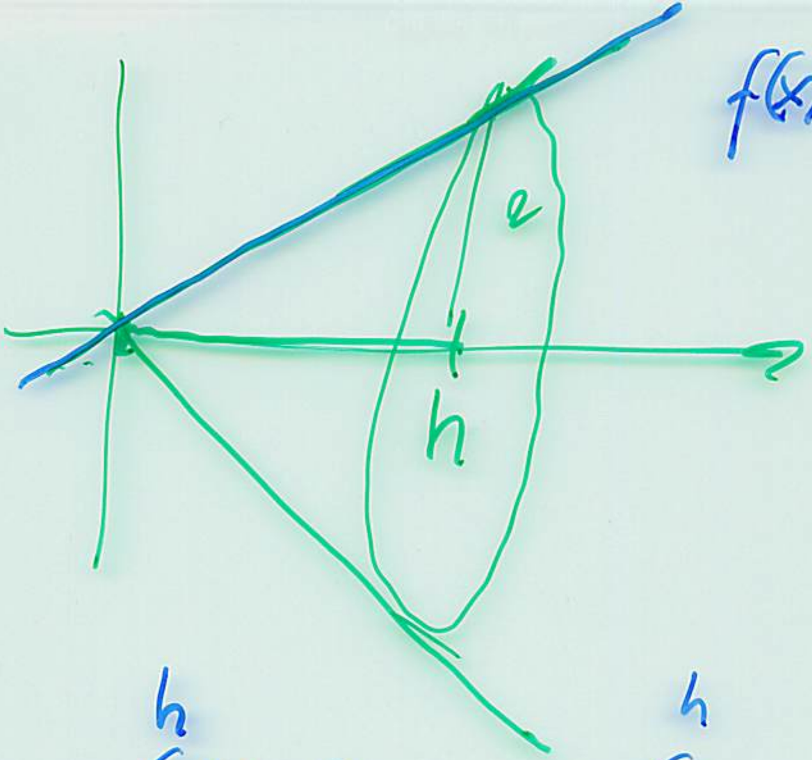




Volume: $(f(x_i))^2 \pi \, \Delta x$

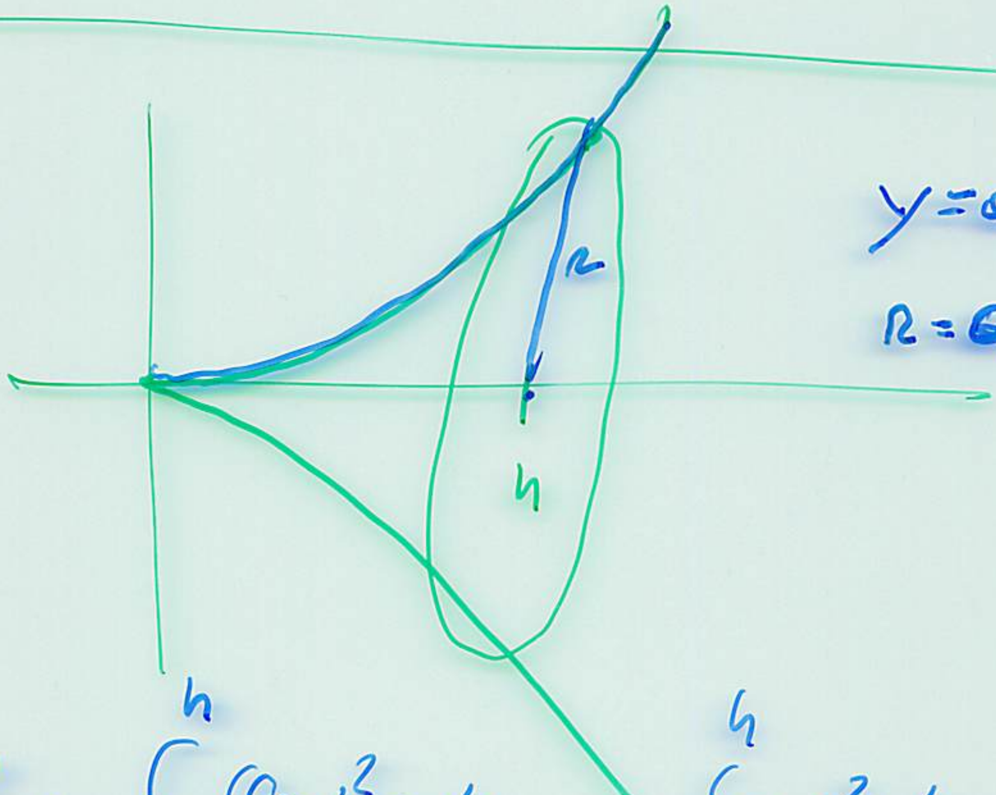
$$\text{Volume} = \sum_{i=1}^{n} \underbrace{(f(x_i))^2 \pi}_{g(x_i)} \, \Delta x_i \longrightarrow \int_a^b (f(x))^2 \pi \, dx$$

Bspl



$$f(x) = ax = \frac{R}{h}x$$

$$V = \int_0^h \left(f(x)\right)^2 \pi \, dx = \int_0^h \pi \frac{R^2}{h^2} x^2 \, dx = \pi \frac{R^2}{h^2} \frac{h^3}{3} = \boxed{\frac{\pi R^2 h}{3}}$$

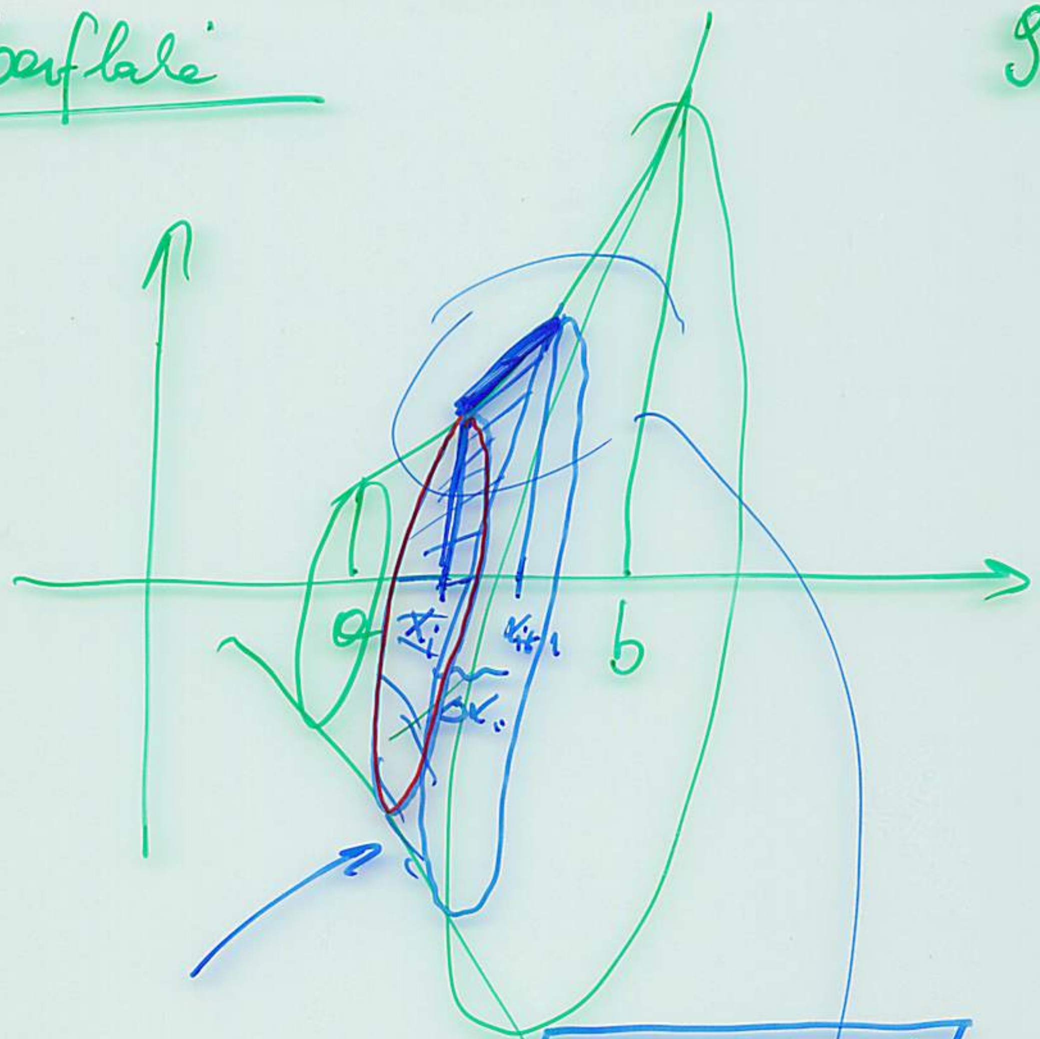


$$y = ax^2$$

$$R = ah^2 \Rightarrow a = \frac{R}{h^2}$$

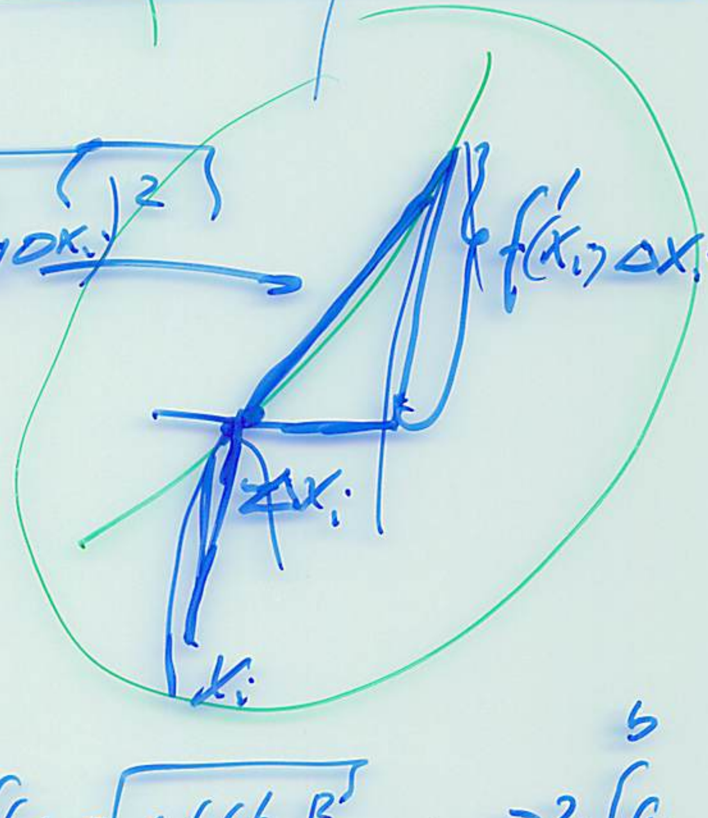
$$V = \int_0^h \left(f(x)\right)^2 \pi \, dx = \int_0^h \pi \frac{R^2}{h^4} x^4 \, dx = \boxed{\frac{\pi R^2 h}{5}}$$

Oberfläche



$$\text{Fläche} \approx 2\pi f(x_i) \cdot \sqrt{1 + (f'(x_i))^2}\, \Delta x_i$$

$$\sqrt{(\Delta x_i)^2 + (f'(x_i)\Delta x_i)^2}$$



$$\text{Gesamtfl} = \sum_{i=1}^{n} \underbrace{2\pi f(x_i)\sqrt{1+(f'(x_i))^2}}_{g(x_i)}\, \Delta x_i \to 2\pi \int_a^b f(x)$$

$$\longrightarrow 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$$

Bspl ~~Zylinder~~ Kegel

$$f(x) = \frac{r}{h}x$$

h

$$f'(x) = \frac{r}{h}$$

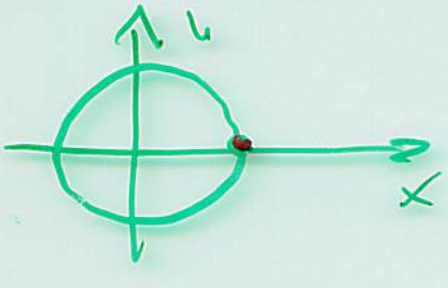
$$\text{Mantel}_{\text{Kegel}} = 2\pi \int_0^h \frac{r}{h}x\sqrt{1+\left(\frac{r}{h}\right)^2}\,dx$$

$$= \pi r h\sqrt{1+\left(\frac{r}{h}\right)^2} = \pi r \cdot \sqrt{h^2+r^2}$$

Kurven

a) $c_1(t) = x(t) = r\cos t$ $\quad t \in [0, 2\pi]$

$c_2(t) = y(t) = r\sin t$



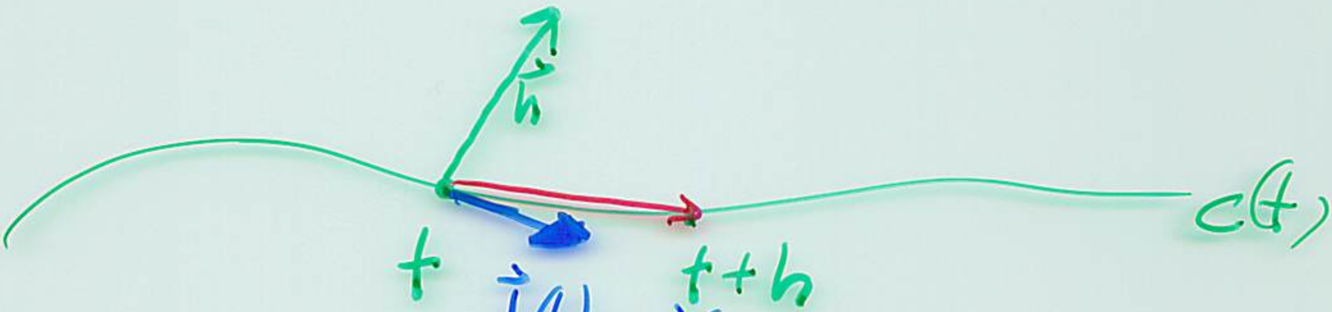
geschlossene Ebene ~~Kur~~ C^1-Kurve

(C^2)

(C^∞)

$$\dot{c}(t) = r(-\sin t, \cos t) \neq 0$$

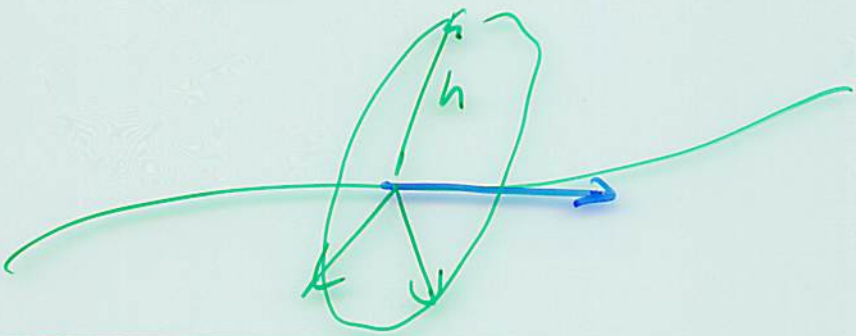
glatt



$$\vec{t}(t) = \dot{c}(t)$$

$$\lim_{h \to 0} \frac{c(t+h) - c(t)}{h} = \dot{c}(t)$$

$$\vec{n} \cdot \vec{t} = \underline{\underline{\vec{n}(t) \cdot \dot{c}(t) = 0}}$$



※ $c_1(t) = a \cos t \,(1 + \cos t)$

$c_2(t) = a \sin t \,(1 + \cos t)$ $\quad t \in [0, 2\pi]$

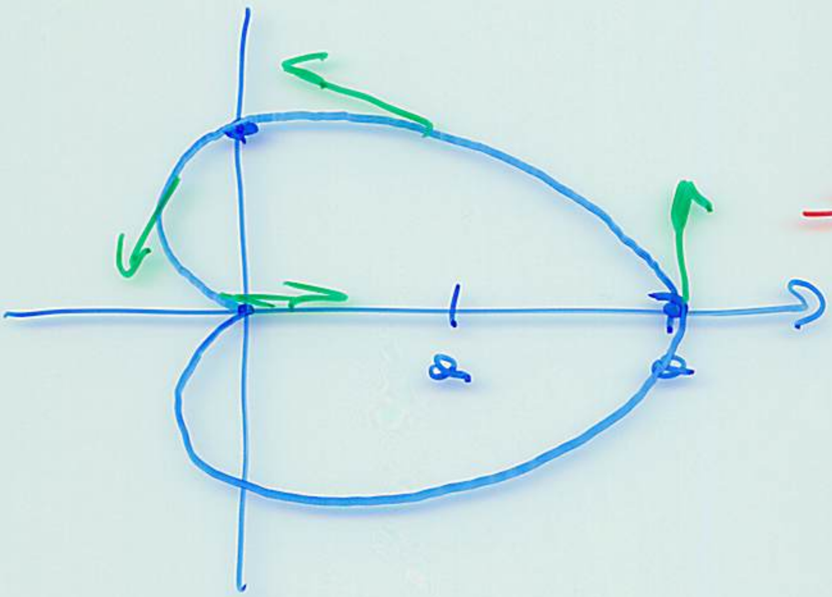
Polarkoordinaten

$$c_1 = R(\varphi) \cos \varphi$$
$$c_2 = R(\varphi) \sin \varphi$$

$R(\varphi) = r\,(1 + \cos \varphi)$.

$r = a$
$\varphi = t$

Cardioide



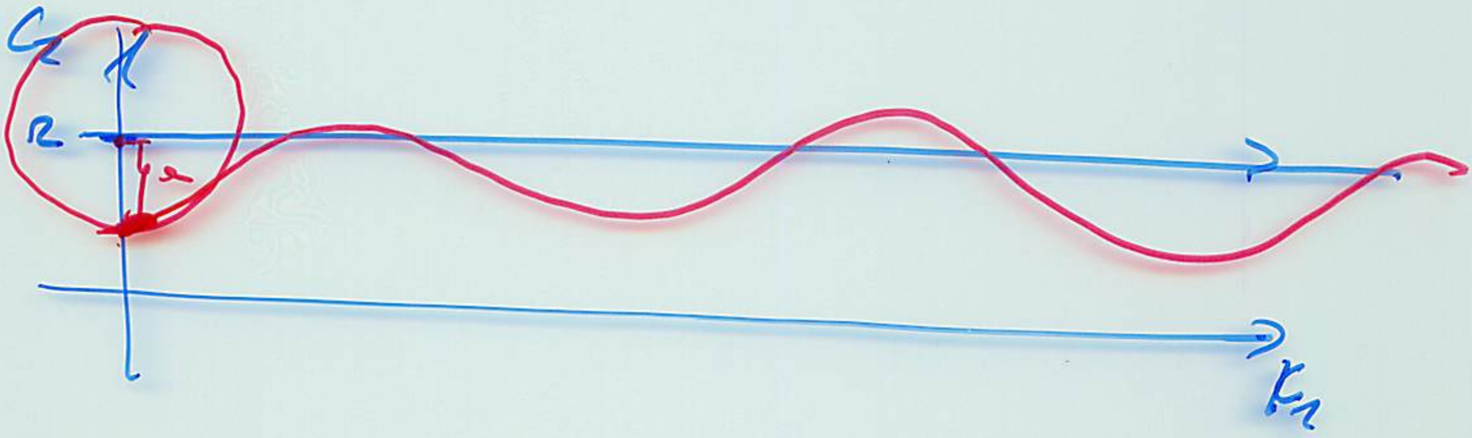
$$\dot c_1 = -a \sin t\,(1+\cos t) + a \cos t\,(1-\sin t)$$
$$\dot c_2 = a \cos t\,(1+\cos t) + a \sin t\,(1-\sin t)$$

$$(\dot c_1, \dot c_2)(t=\pi) = \_\_\_\_\_$$

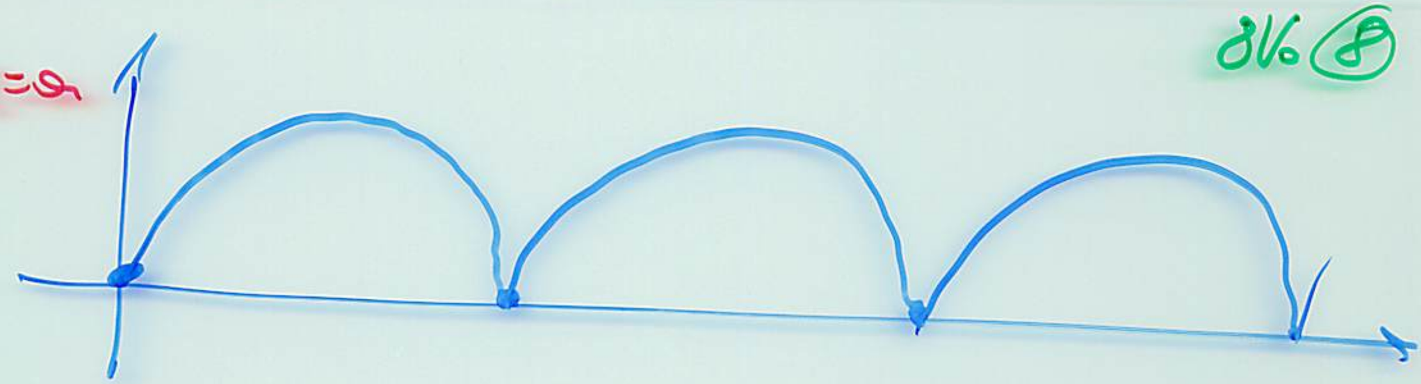
Zykloide

$$c(t) = (Rt - a \sin t,\ R - a \cos t)$$

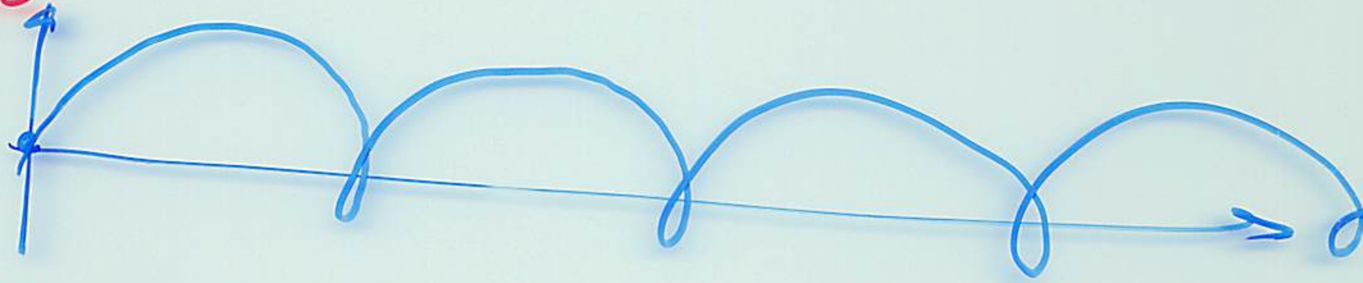
$$= R(t,1) - a(\sin t, \cos t)$$



$R = a$

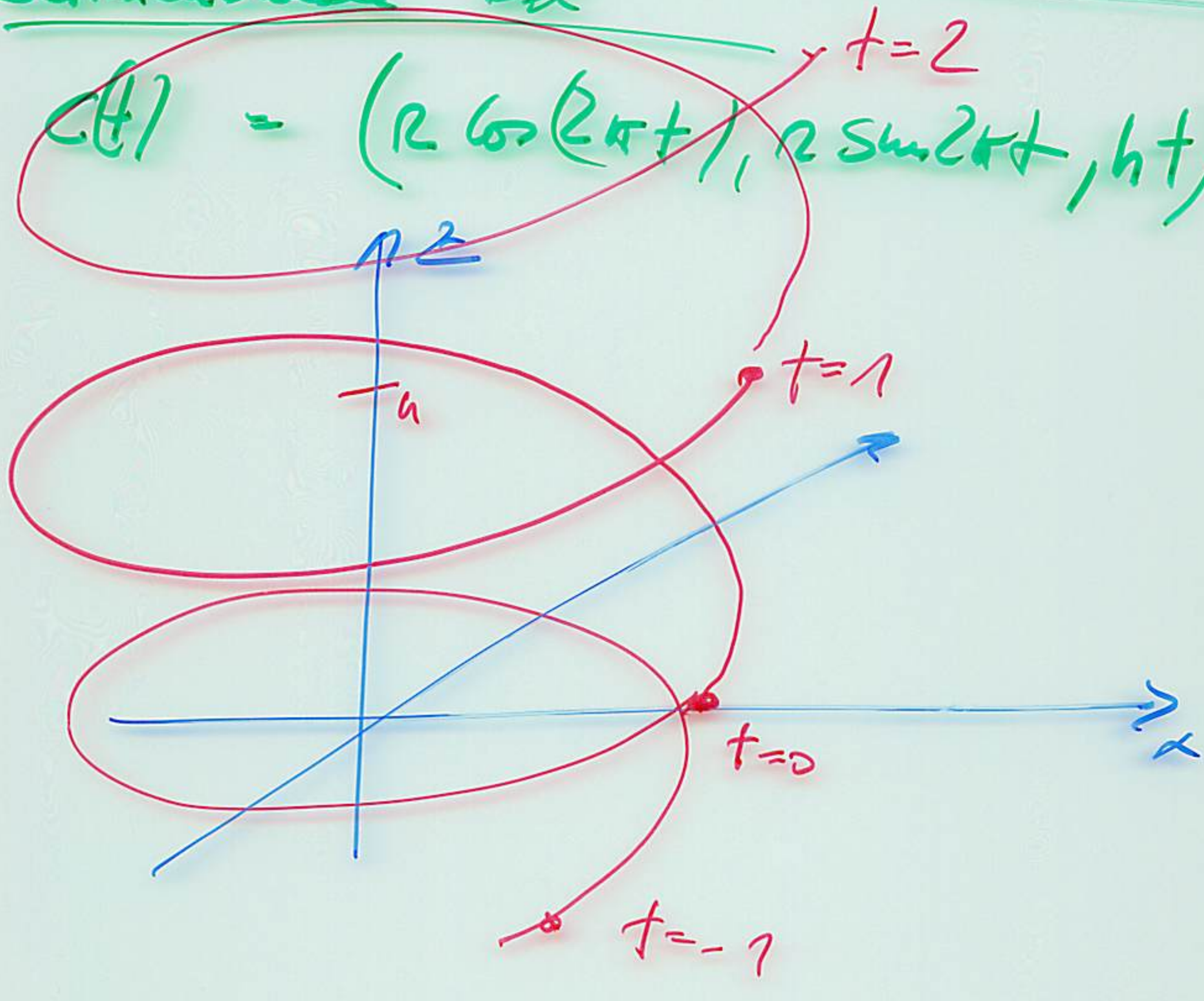


$a > R$



3dimensionale Bsp

$$c(t) = (R\cos(2\pi t), R\sin 2\pi t, ht)$$



$c : [a,b] \to \mathbb{R}^n$

$h : [\alpha,\beta] \to [a,b]$

$\tilde{c}(\tau) = c(h(\tau))$

$c(t) = r(\cos t, \sin t) \quad t \in [0, 2\pi]$

$h(\tau) = 3\tau$

$h : [\alpha = 0, \beta = \frac{2}{3}\pi] \to [0, 2\pi]$

$\tilde{c}(\tau) = c(h(\tau)) = \underline{r(\cos 3\tau, \sin 3\tau)}$

Vorsicht

$h(\tau) = e^\tau$

$h : [\alpha,\beta] \not\to [0, 2\pi]$

$h(\tau) = e^\tau - 1$

$h : [0, \ln(2\pi + 1)] \to [0, 2\pi]$