Vo An III 3.2.2010

$$\int_K f(x)\,dx$$

HI. 1. Art $\quad \int_C f(x)\,ds = \int_a^b f(c(t))\,\|\dot c(t)\|\,dt$

2. Art $\quad \int_C f(x)\,dx = \int_a^b \langle f(c(t)), \dot c(t)\rangle\,dt = \int_C \langle f, T\rangle\,ds$

OI. 1. Art $\quad \int_K f(x)\,d\sigma = \int_U f(p(u))\,\left\|\frac{\partial p}{\partial u_1} \times \frac{\partial p}{\partial u_2}\right\|\,du$

2. Art $\quad \int_K f(x)\,d\sigma = \int \langle f(p(u)), n(p(u))\rangle \|\quad\|\,du$

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2d Green: $\quad \oint_{\partial K} f\,dx = \int_K \operatorname{rot} f\,dx \quad K$

3d Green:

2d Gauss $\quad \oint_{\partial K} \langle f, n\rangle\,ds = \int_K \operatorname{div} f\,dx$

3d Gauss $\quad \int_{\partial K} \langle f, n\rangle\,d\sigma = \int_K \operatorname{div} f\,dx$

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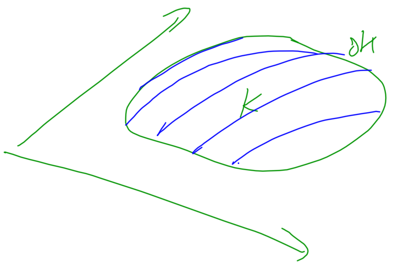

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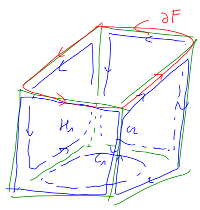

$$\int_{\partial F} f\,dx = \int_{C_1} + \int_{C_2} + \dots + \int_{C_5}$$

5× Green

$$= \int_{K_1} \operatorname{rot}_{2d} f\,dx + \dots\dots + \int_{K_5} \operatorname{rot} f\,dx$$

$$= \int_{K_1} \langle \operatorname{rot}_{3d} f, n\rangle\,d\sigma + \dots\dots \int_{K_5}$$

$$= \int_K \langle \operatorname{rot} f, n\rangle\,d\sigma$$

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Beispiel:

Gegeben sei das Vektorfeld

$$\mathrm{f}(x, y, z) = (-y, x, -z)^T$$

und die geschlossene Kurve $\mathrm{c} : [0, 2\pi] \to \mathbb{R}^3$ parametrisiert durch

$$\mathrm{c}(t) = (\cos t, \sin t, 0)^T, \quad 0 \le t \le 2\pi$$

Dann gilt:

$$\oint_c \mathrm{f}(\mathrm{x})\,d\mathrm{x} = \int_0^{2\pi} \langle \mathrm{f}(\mathrm{c}(t)), \dot{\mathrm{c}}(t)\rangle\,dt$$

$$= \int_0^{2\pi} \left\langle \begin{pmatrix} -\sin t \\ \cos t \\ 0 \end{pmatrix}, \begin{pmatrix} -\sin t \\ \cos t \\ 0 \end{pmatrix} \right\rangle dt$$

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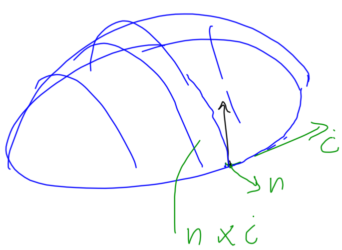

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