Fachbereich Mathematik der Universität Hamburg **WiSe 2021/22**
Prof. Dr. I. Gasser
Dr. H. P. Kiani, S. Onyshkevych

Analysis III for Engineering Students

Work sheet 2

Exercise 1: Let $f, g : \mathbb{R}^2 \longrightarrow \mathbb{R}$.

$$f(x,y) := 3x - 5y, \qquad g(x,y) := \frac{1}{5}(x^2+y^2) + 1\,.$$

a) Calculate the gradients of f and g.

b) For f draw the contour lines (level curves)
$f^{-1}(C) := \{(x,y)^{\mathrm{T}} : f(x,y) = C\}$
for the function values $C_1 = 5, C_2 = 0$ and $C_3 = -10$.
At points $P_1 = \binom{0}{-1}, P_2 = \binom{5}{3}$ and $P_3 = \binom{-5}{-1}$ also provide the direction of the gradient.

c) For g draw the contour lines
$g^{-1}(C) := \{(x,y)^{\mathrm{T}} : g(x,y) = C\}$
for function values $C_4 = \frac{6}{5}, C_5 = \frac{21}{5}$ and $C_6 = 6$.
At points $P_4 = \binom{0}{-1}, P_5 = \binom{4}{0}$ and $P_6 = \binom{3}{4}$ also provide the direction of the gradient.

d) Based on your observations (i.e. without proof), try to formulate a guess on how the direction of gradient at a given point is related to the direction of the contour line through that point.

Exercise 2:

Let

$$f : \mathbb{R}^2 \to \mathbb{R}\,, \qquad f(x,y) = \cos(2x-3y) + x^3 - y^3 + 2y^2\,.$$

a) Find all first, second and third order partial derivatives of f.

b) the *tangential plane* to the graph of a differentiable function $f : D_f \longrightarrow \mathbb{R}$ at point $(x^0, y^0) \in D_f \subset \mathbb{R}^2$ is defined by

$$z = f(x^0,y^0) + f_x(x^0,y^0)\,(x-x^0) + f_y(x^0,y^0)\,(y-y^0)\,.$$

Give the equation of the tangential plane to the graph of f at the point $(x^0, y^0) = (\frac{\pi}{4}, 0)$.

Classes: 01.–05.11.21