Fachbereich Mathematik der Universität Hamburg **WiSe 2021/22**
Prof. Dr. I. Gasser
Dr. H. P. Kiani, S. Onyshkevych

Analysis III for Engineering Students

Work sheet 4

Exercise 1:

a) Find an approximation to a local minimum of the function

$$f \; : \; \left[-\frac{1}{4}\,,\,\frac{1}{4}\right] \times \left[-\frac{1}{4}\,,\,\frac{1}{4}\right] \to \mathbb{R}$$

$$f(x,y) \;=\; 4x^2 + xy + 4y^2 + \sin(x-y)\,,$$

by computing the minimum $\begin{pmatrix}\tilde{x}\\ \tilde{y}\end{pmatrix}$ of the second-degree Taylor polynomial T_2 of f centered at the point $(0,0)^{\mathrm{T}}$.

Hint: use the sine-series.

b) Estimate the value of the remainder R_2 in the Lagrange form at the already computed point $\begin{pmatrix}\tilde{x}\\ \tilde{y}\end{pmatrix}$.

Hint: One does not need to compute every derivative exactly.

c) Show that the minimum value of f, on the domain specified above, can not be smaller than $-\frac{9}{49}$.

Exercise 2: Determine the stationary points of the following functions and check whether they are minima, maxima or saddle points:

a) $f(\boldsymbol{x}) := \boldsymbol{x}^T \boldsymbol{A}\, \boldsymbol{x} \;+\; \boldsymbol{b}^T \boldsymbol{x} \;+ c$ with

$$\boldsymbol{x} \;:=\; \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2\,, \qquad \boldsymbol{A} \;:=\; \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}\,, \qquad \boldsymbol{b} \;:=\; \begin{pmatrix} -4 \\ 12 \end{pmatrix}\,, \qquad c = 2018\,,$$

b) $g(x,y) := x^2 - xy - x + \frac{y^4}{4} + \frac{y^3}{3}$.

Discussion: 29.11 – 03.12.21