Fachbereich Mathematik der Universität Hamburg **WiSe 2021/22**
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Analysis III for Engineering Students

Homework sheet 5

Exercise 1

The equation

$$g(x, y) = \left(x^2 + 4y^2\right)^2 + x^2 - 4y^2 = 0$$

is an implicit description of the curve in $\mathbb{R}^2$.

a) Show that $(x, y) = (0, 0)^T$ is a singular point of the implicitly defined curve

$$\left(x^2 + 4y^2\right)^2 + x^2 - 4y^2 = 0$$

and determine whether it is an isolated point, double point or a return point (cusp).

b) Show that there are no other singular points.

c) Compute the points on the curve with horizontal or vertical tangent.

Exercise 2: We are looking for the extrema of the function

$$f(x, y) = 2 \ln\left(\frac{x}{y}\right) + x + 5y$$

that fulfill the constraint

$$g(x, y) = x\,y - 1 = 0.$$

a) Show that $(x_0, y_0)^T = (1, 1)^T$ with the suitable fixed λ is a feasible stationary point of the Lagrangian $F = f + \lambda g$ and check the regularity conditions at the point $(x_0, y_0)^T = (1, 1)^T$.

b) Determine of what type the stationary point $(x_0, y_0)^T = (1, 1)^T$ is. To do so, assemble the Hessian matrix $\boldsymbol{H_x} F(x_0, y_0)$ and check its definiteness on the tangent space $\ker\left(Dg(x_0, y_0)\right)$.

Exercise 3)

Compute

a) the integral

$$\int\int_{D_1} xy^2 d(x,y) \quad , \text{ where } D_1 = [-1,3] \times [1,2],$$

b) the volume of the body $K \subset \mathbb{R}^3$,

$$K = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| \; |x| \le 1, \;\; -(1-x^2) \le y \le 1-x^2, \;\; 0 \le z \le \left(1 - x^2 - y\right) \right\},$$

c) and the integral

$$\int\int_{D_2} (x^2 - y^4) d(x,y) \quad , \text{ where } D_2 = \{(x,y) : |x| + |y| \le 1\}.$$

Hint: Use the symmetries!

Submission deadline: 13.12.–17.12.21