Fachbereich Mathematik der Universität Hamburg **WiSe 2021/22**
Prof. Dr. I. Gasser
Dr. H. P. Kiani, S. Onyshkevych

Analysis III for Engineering Students

Homework sheet 6

Exercise 1:

a) Given a set

$$D := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \, : \, \frac{y^2}{2} - 2 \, \leq \, x \, \leq \, 4 - y^2 \right\}$$

Sketch the set D and determine the center of mass of D with the uniform mass density (mass/unit area) $\rho = 2$.

Hint: It holds

Mass: $M = \int_D \rho(\boldsymbol{x}) d\boldsymbol{x}$

Center of mass: $X_s = \frac{1}{M} \int_D \rho(\boldsymbol{x}) \, \boldsymbol{x} \, d\boldsymbol{x}$ (componentwise)

b) Let $K := \left\{ (x, y, z)^T \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1, \, z \geq 0 \right\}$. Compute

$$\int_K (y^2 - x^2) \, d(x, y, z)$$

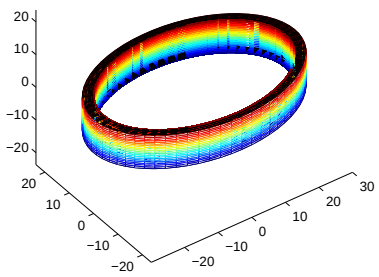
Hint:
- *To reduce the amount of work one can use* **spherical coordinate system**.
- *It holds* $\cos(2t) = \cos^2(t) - \sin^2(t)$.

Exercise 2:

Given is the elliptical pipe section

$$R \subset \mathbb{R}^3, \quad R : \, 81 \leq \left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 \leq 100, \quad -5 \leq z \leq 5 \, .$$

The piece of pipe has the constant density ρ.



Compute the volume, mass and moment of inertia of the pipe section with respect to the $y-$ axis using integration. Use elliptical cylindrical coordinates

$$x = 3r\cos(\varphi)\,,\; y = 2r\sin(\varphi)\,,\; z = z\,.$$

Hint:

$\cos^2(\phi) = \frac{\cos(2\phi)+1}{2}$.

Since we do not use a calculator, there is no need to calculate the precise final value. It is sufficient to only insert the integration limits into the calculated root functions.

Submission deadline: 10.01.–14.01.22